http://poloclub.gatech.edu/cse6242

CSE6242 / CX4242: Data & Visual Analytics

Time Series

Mining and Forecasting

Duen Horng (Polo) Chau
Assistant Professor
Associate Director, MS Analytics
Georgia Tech

Partly based on materials by
Professors Guy Lebanon, Jeffrey Heer, John Stasko, Christos Faloutsos, Parishit Ram (GT PhD alum; SkyTree), Alex Gray

Outline

- Motivation
- Similarity search – distance functions
- Linear Forecasting
- Non-linear forecasting
- Conclusions

Problem definition

- **Given**: one or more sequences

  $x_1 , x_2 , \dots , x_t , \dots$

  $(y_1, y_2, \dots , y_t , \dots)$

  $(\dots)$
- **Find**
  - similar sequences; forecasts
  - patterns; clusters; outliers

Motivation - Applications

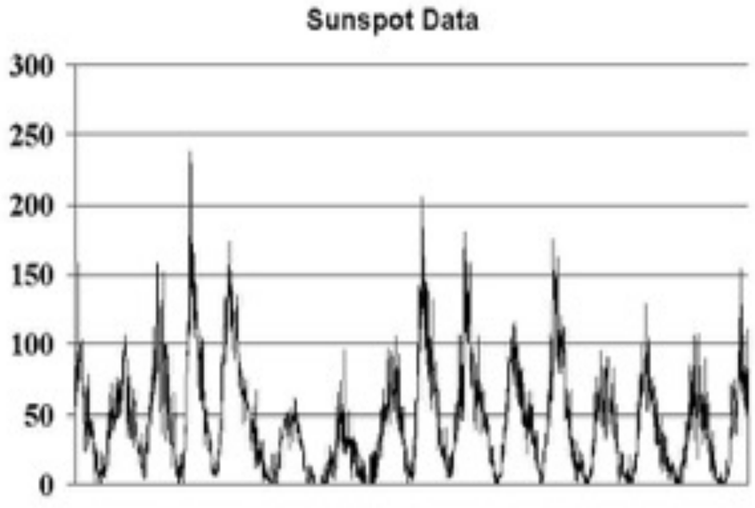
- Financial, sales, economic series
- Medical
  - ECGs +; blood pressure etc monitoring
  - reactions to new drugs
  - elderly care

Motivation - Applications (cont'd)

- 'Smart house'
  - sensors monitor temperature, humidity, air quality
- video surveillance

Motivation - Applications (cont'd)

- Weather, environment/anti-pollution
  - volcano monitoring
  - air/water pollutant monitoring

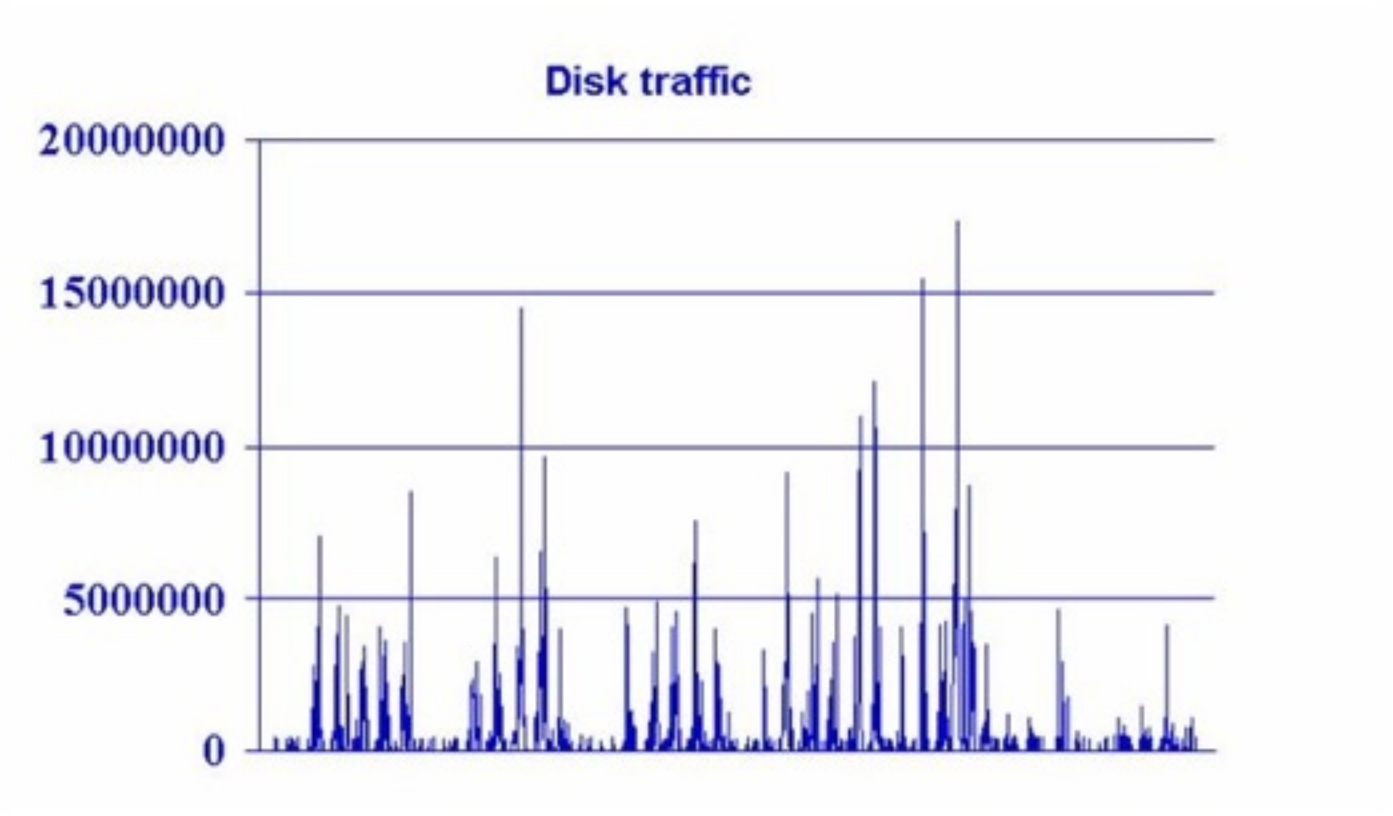


Motivation - Applications (cont’d)

- Computer systems
  - ‘Active Disks’ (buffering, prefetching)
  - web servers (ditto)
  - network traffic monitoring
  - ...

Stream Data: Disk accesses

#bytes



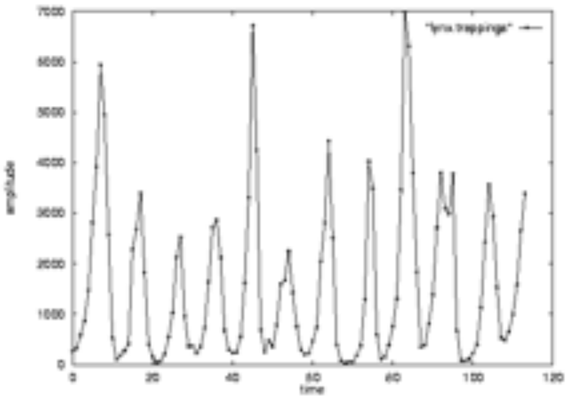
time

Problem #1:

Goal: given a signal (e.g.., #packets over time)

Find: patterns, periodicities, and/or compress

count

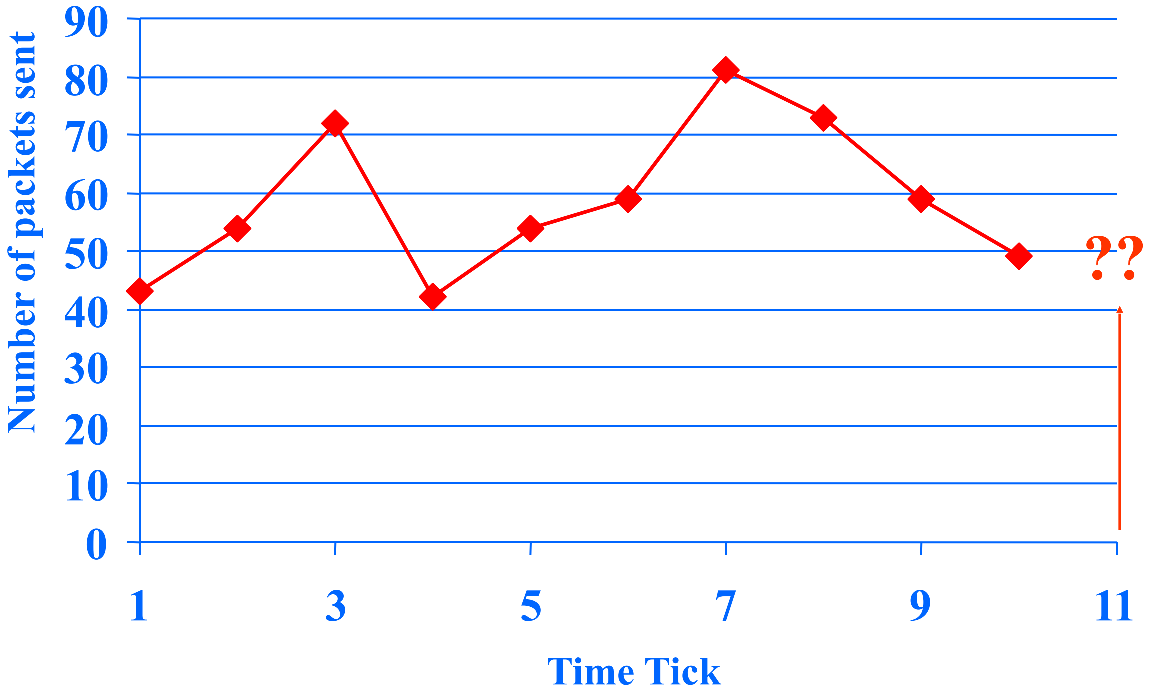


year

lynx caught per year
(packets per day;
temperature per day)

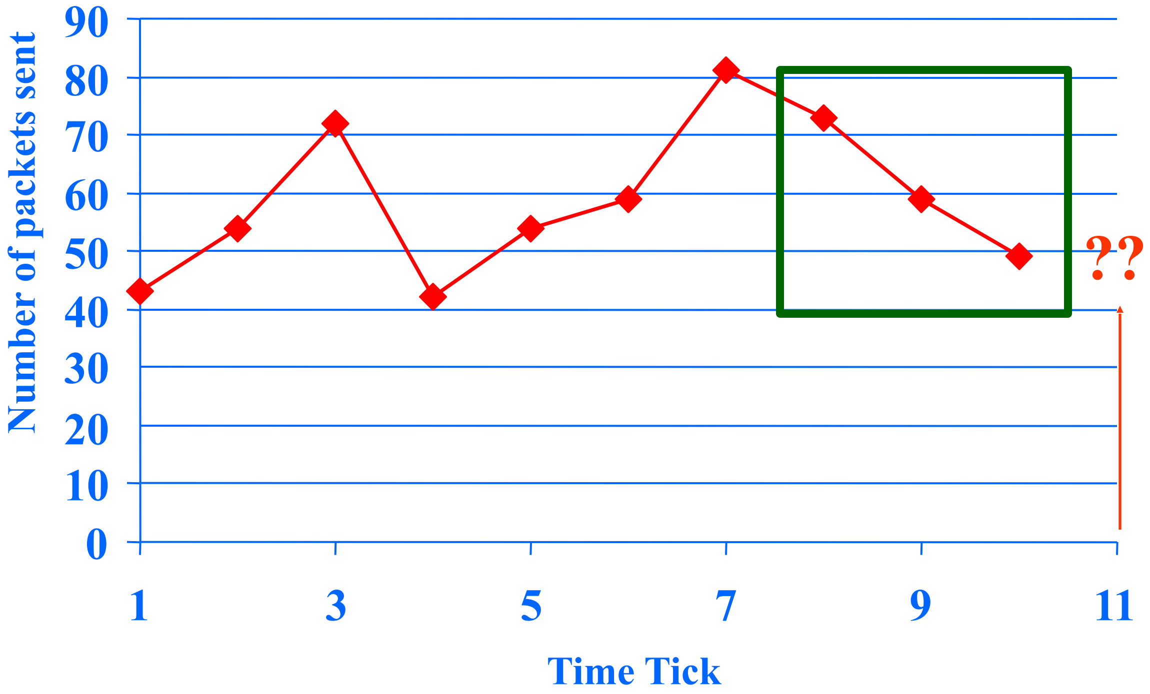
Problem#2: Forecast

Given x_t, x_{t-1}, …, forecast x_{t+1}



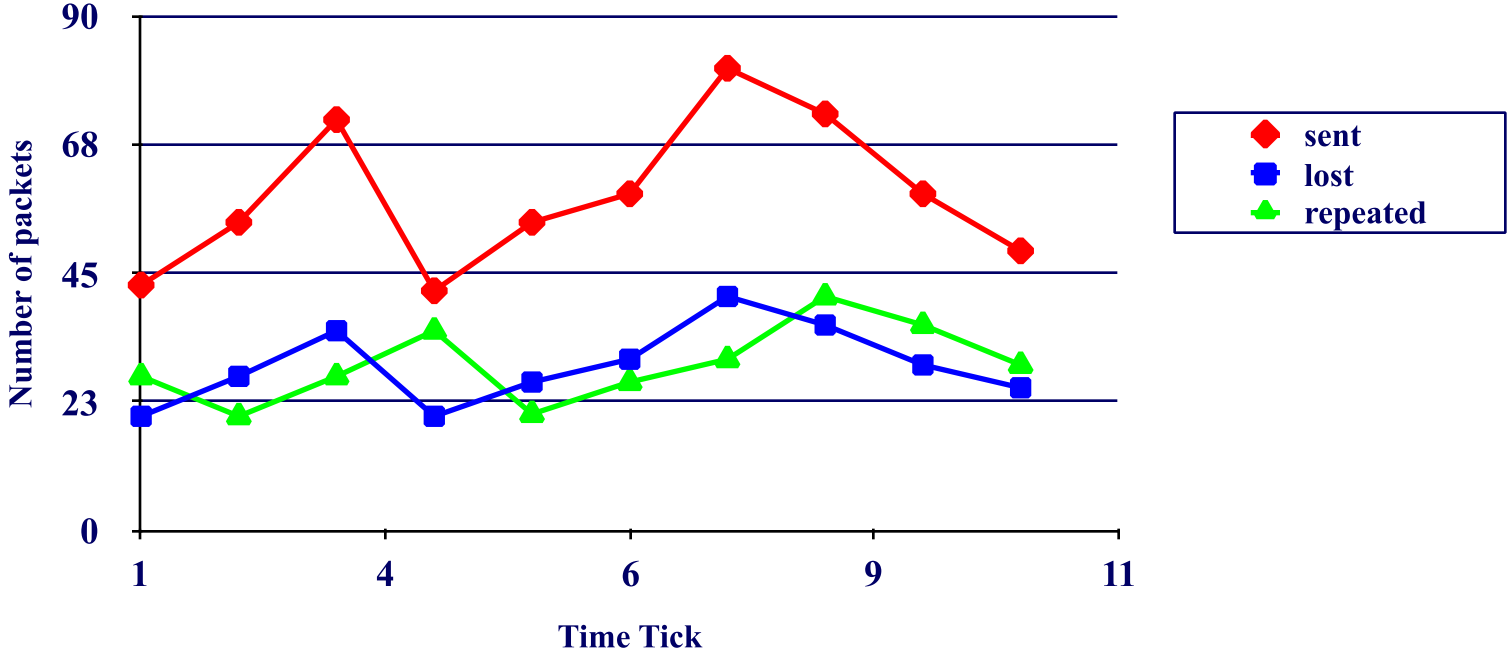
Problem#2’: Similarity search

E.g.., Find a 3-tick pattern, similar to the last one



Problem #3:

- Given: A set of **correlated** time sequences
- Forecast 'Sent(t)'



Important observations

Patterns, rules, forecasting and similarity indexing are closely related:

- To do forecasting, we need
  - to find patterns/rules
  - to find similar settings in the past
- to find outliers, we need to have forecasts
  - (outlier = too far away from our forecast)

Outline

- Motivation
- Similarity search and distance functions
  - Euclidean
  - Time-warping
- …

Importance of distance functions

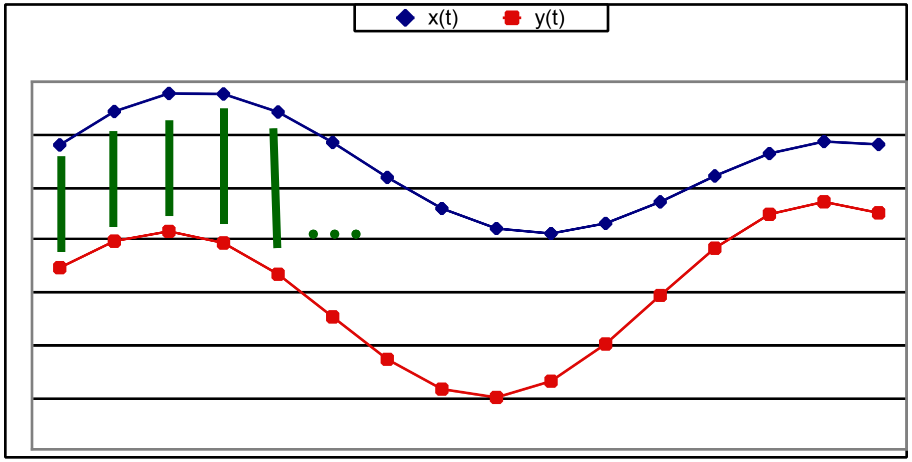
Subtle, but **absolutely necessary**:

- A ‘must’ for similarity indexing (-> forecasting)
- A ‘must’ for clustering

Two major families

- – Euclidean and Lp norms
- – Time warping and variations

Euclidean and Lp



$$D(\vec{x},\vec{y}) = \sum_{i=1}^{n}(x_i - y_i)^2$$

$$L_p(\vec{x},\vec{y}) = \sum_{i=1}^{n}|x_i - y_i|^p$$

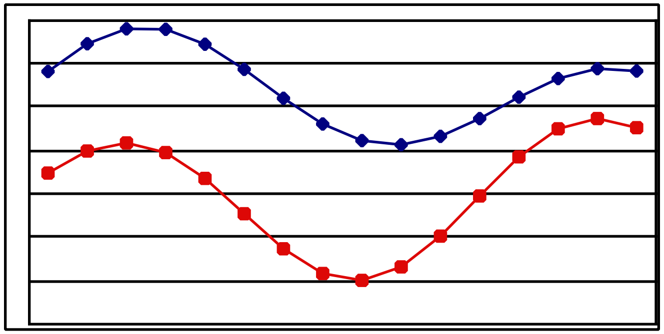
L_1: city-block = Manhattan

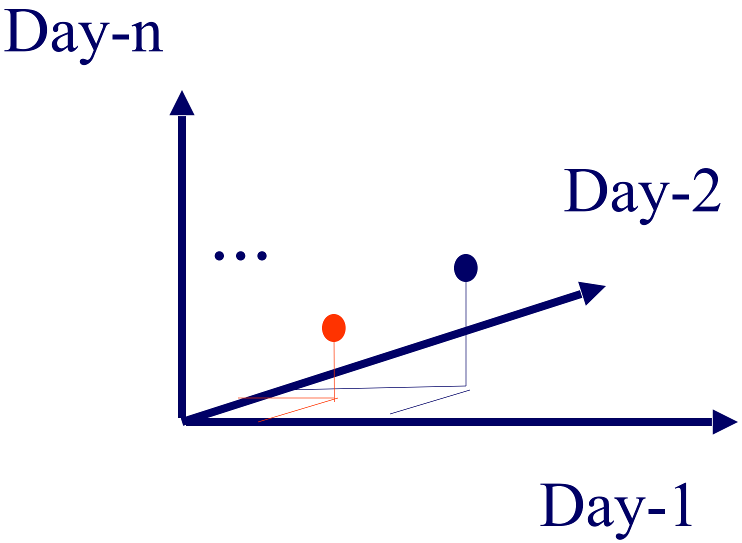
L_2 = Euclidean

L_∞

Observation #1

Time sequence -> n-d vector

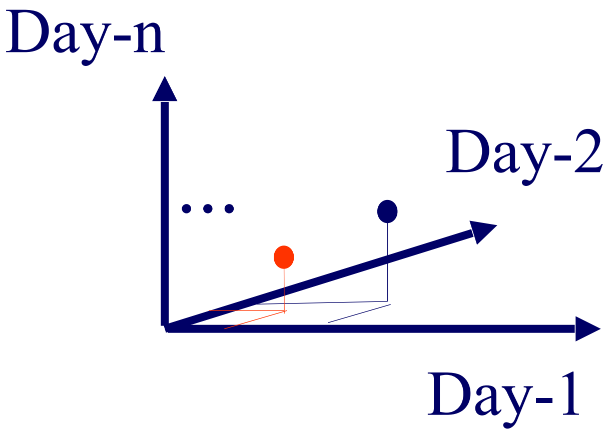


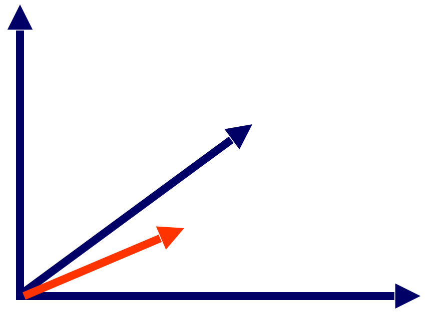


Observation #2

Euclidean distance is closely related to

- cosine similarity
- dot product
- ‘cross-correlation’ function

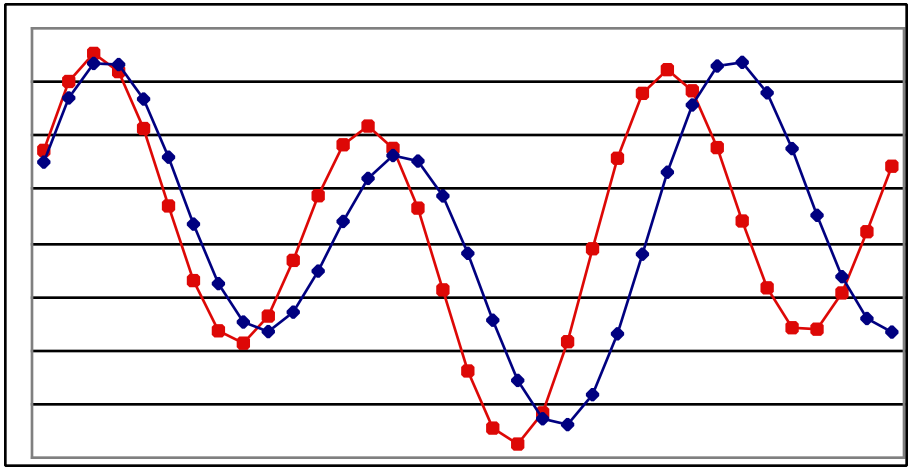




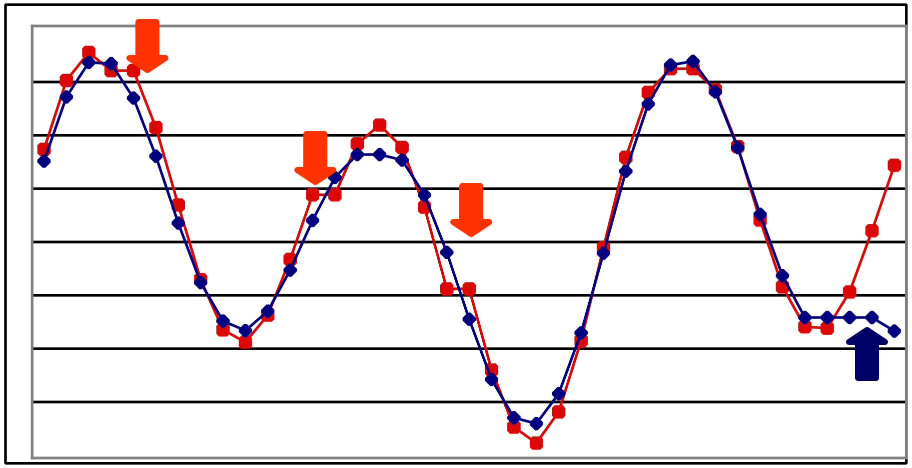
Time Warping

- allow accelerations - decelerations
  - (with or without penalty)
- THEN compute the (Euclidean) distance (+ penalty)
- related to the string-editing distance

Time Warping



‘stutters’:



Time warping

Q: how to compute it?

A: dynamic programming

$D(i, j)$ = cost to match

prefix of length i of first sequence x with prefix of length j of second sequence y

Time warping

Thus, with no penalty for stutter, for sequences

$$x_1, x_2, \ldots, x_{i,;} \qquad y_1, y_2, \ldots, y_j$$

$$D(i,j) = \left\| x[i] - y[j] \right\| + \min \begin{cases} D(i-1, j-1) & \text{no stutter} \\ D(i, j-1) & \text{x-stutter} \\ D(i-1, j) & \text{y-stutter} \end{cases}$$

Time warping

VERY SIMILAR to the string-editing distance

$$D(i,j) = \left\| x[i] - y[j] \right\| + \min \begin{cases} D(i-1, j-1) & \text{no stutter} \\ D(i, j-1) & \text{x-stutter} \\ D(i-1, j) & \text{y-stutter} \end{cases}$$

Time warping

- Complexity: O(M*N) - quadratic on the length of the strings
- **Many** variations (penalty for stutters; limit on the number/percentage of stutters; …)
- popular in voice processing [Rabiner + Juang]

Other Distance functions

- piece-wise linear/flat approx.; compare pieces [Keogh+01] [Faloutsos+97]
- ‘cepstrum’ (for voice [Rabiner+Juang])
  - do DFT; take log of amplitude; do DFT again!
- Allow for small gaps [Agrawal+95]

See tutorial by [Gunopulos + Das, SIGMOD01]

Other Distance functions

- In [Keogh+, KDD’04]: parameter-free, MDL based

Conclusions

Prevailing distances:

- Euclidean and
- time-warping

Outline

- Motivation
- Similarity search and distance functions
- Linear Forecasting
- Non-linear forecasting
- Conclusions

Linear Forecasting

Outline

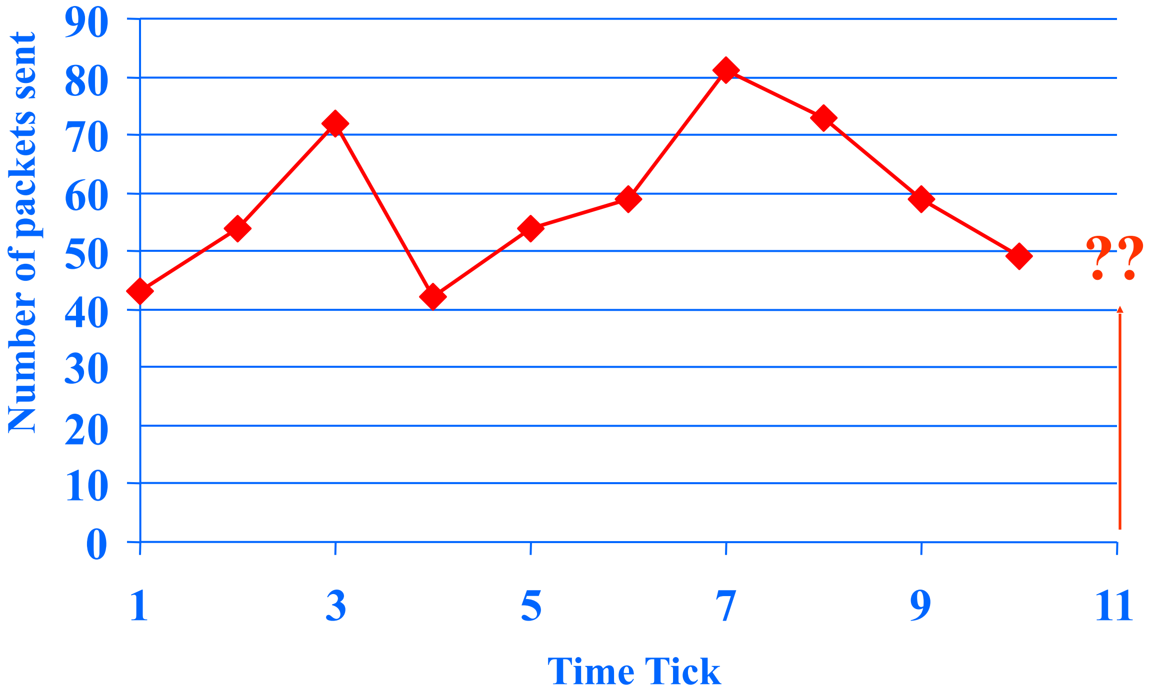
- Motivation
- ...
- Linear Forecasting
  - ➡ Auto-regression: Least Squares; RLS
  - Co-evolving time sequences
  - Examples
  - Conclusions

Reference

[Yi+00] Byoung-Kee Yi et al.: *Online Data Mining for Co-Evolving Time Sequences*, ICDE 2000. (Describes MUSCLES and Recursive Least Squares)

Problem#2: Forecast

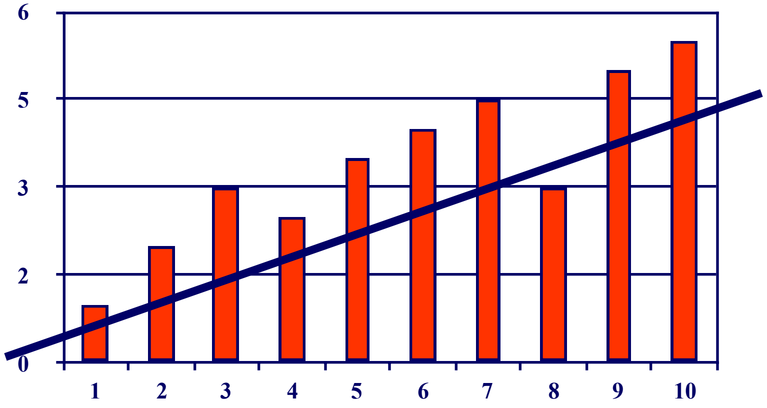
- Example: give x_{t-1}, x_{t-2}, …, forecast x_t



Forecasting: Preprocessing

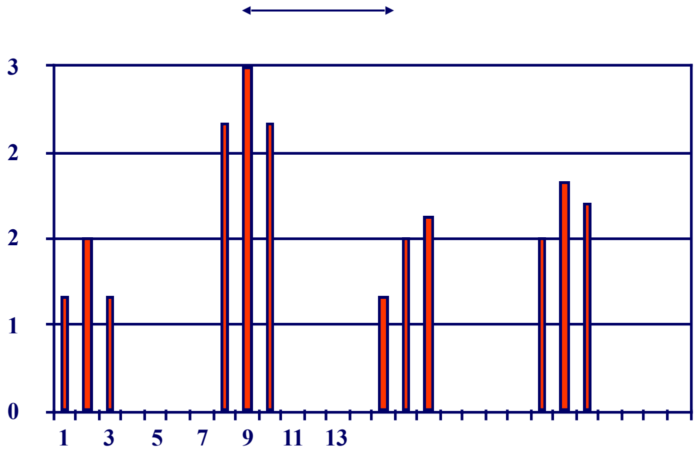
MANUALLY:

remove trends



spot periodicities

7 days



Problem#2: Forecast

- Solution: try to express

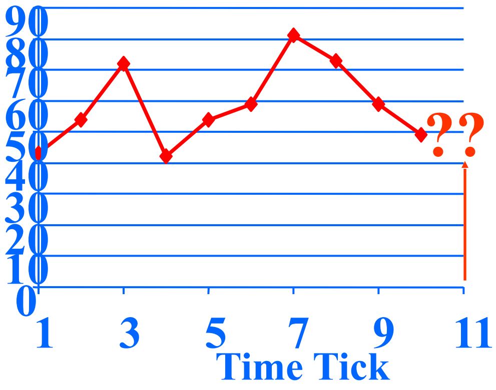
  x_t

  as a linear function of the past: x_{t-1}, x_{t-2}, …,

  (up to a window of w)

Formally:

$$\boxed{x_t \approx a_1 x_{t-1} + \ldots + a_w x_{t-w} + noise}$$



(Problem: Back-cast; interpolate)

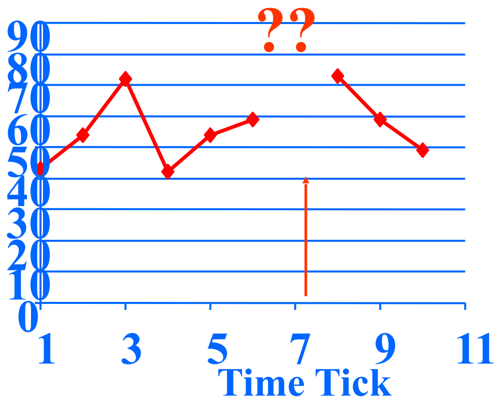
- Solution - interpolate: try to express

  x_t

  as a linear function of the past AND the future:

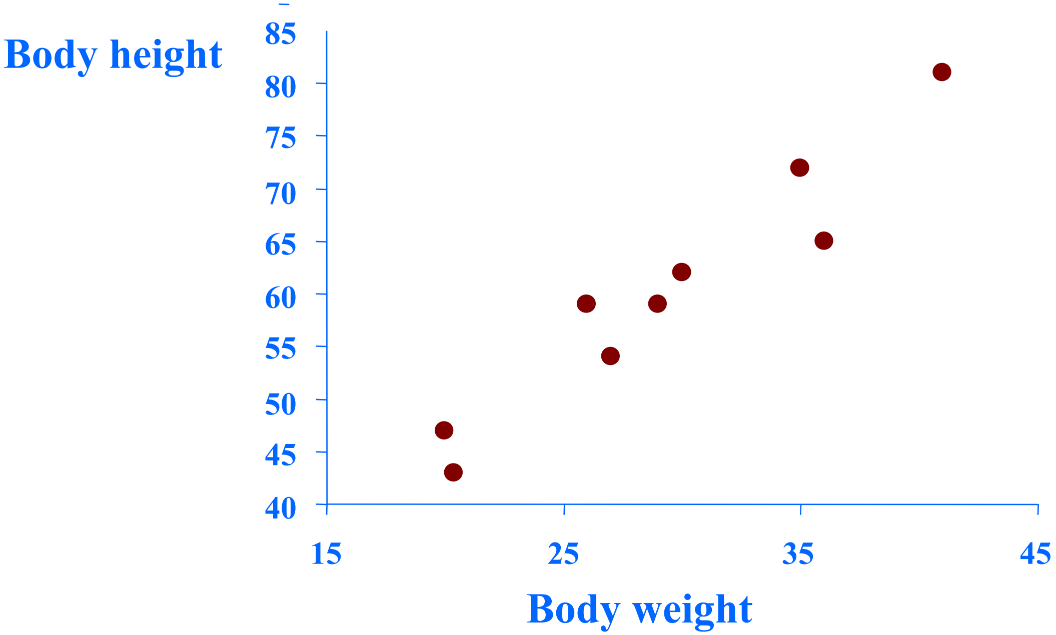
  $x_{t+1}, x_{t+2}, \dots x_{t+wfuture;}\ x_{t-1}, \dots x_{t-wpast}$

  (up to windows of w_{past} , w_{future})
- EXACTLY the same algo's



Refresher: Linear Regression

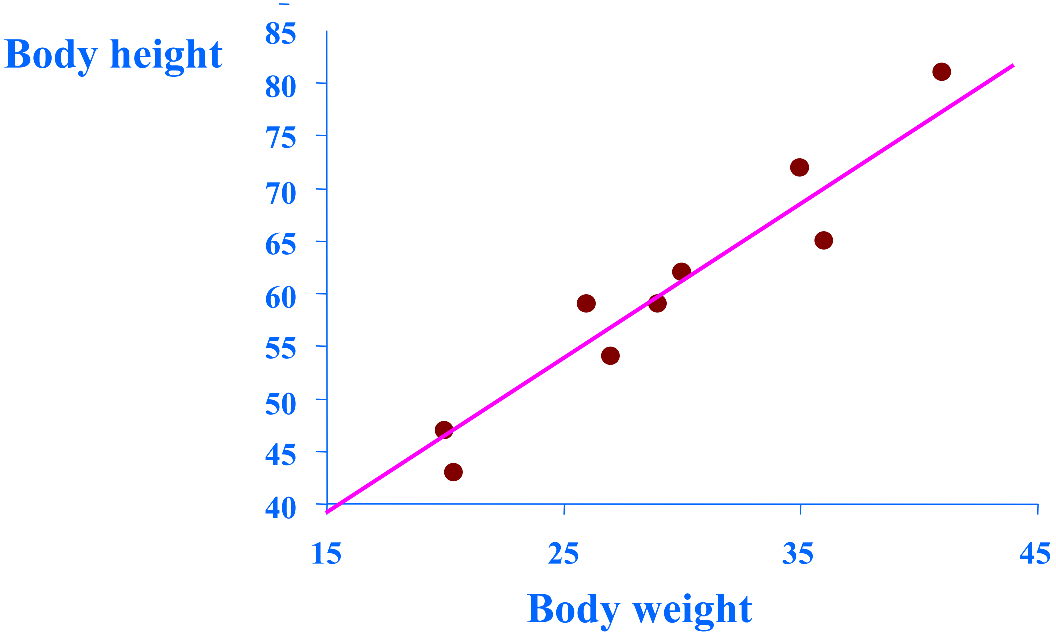
patient	*weight*	*height*
1	27	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



Express what we **don’t know** (= “dependent variable”)
as a linear function of what we **know** (= “independent variable(s)”)

Refresher: Linear Regression

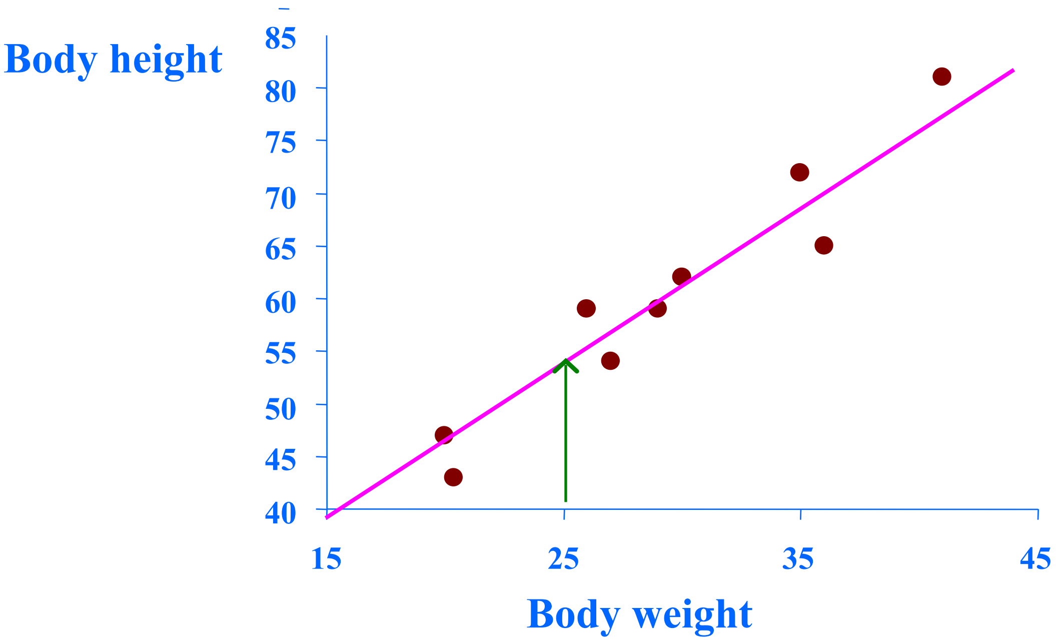
patient	*weight*	*height*
1	27	43
2	43	54
3	54	72
…	…	…
N	**25**	**??**



Express what we **don’t know** (= “dependent variable”)
as a linear function of what we **know** (= “independent variable(s)”)

Refresher: Linear Regression

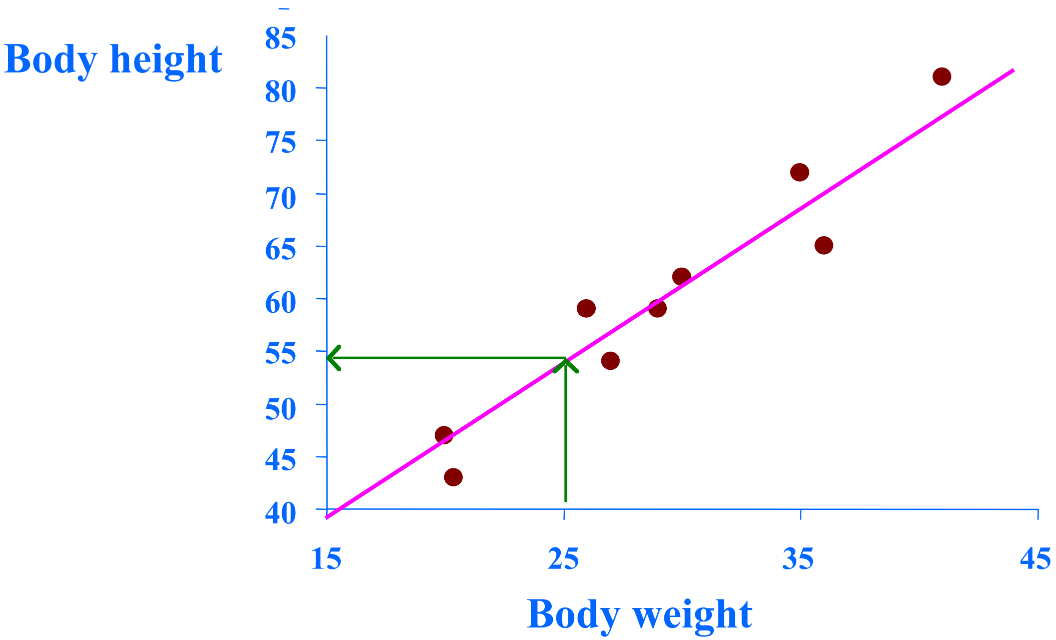
patient	*weight*	*height*
1	27	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



Express what we **don’t know** (= “dependent variable”) as a linear function of what we **know** (= “independent variable(s)”)

Refresher: Linear Regression

patient	*weight*	*height*
1	27	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



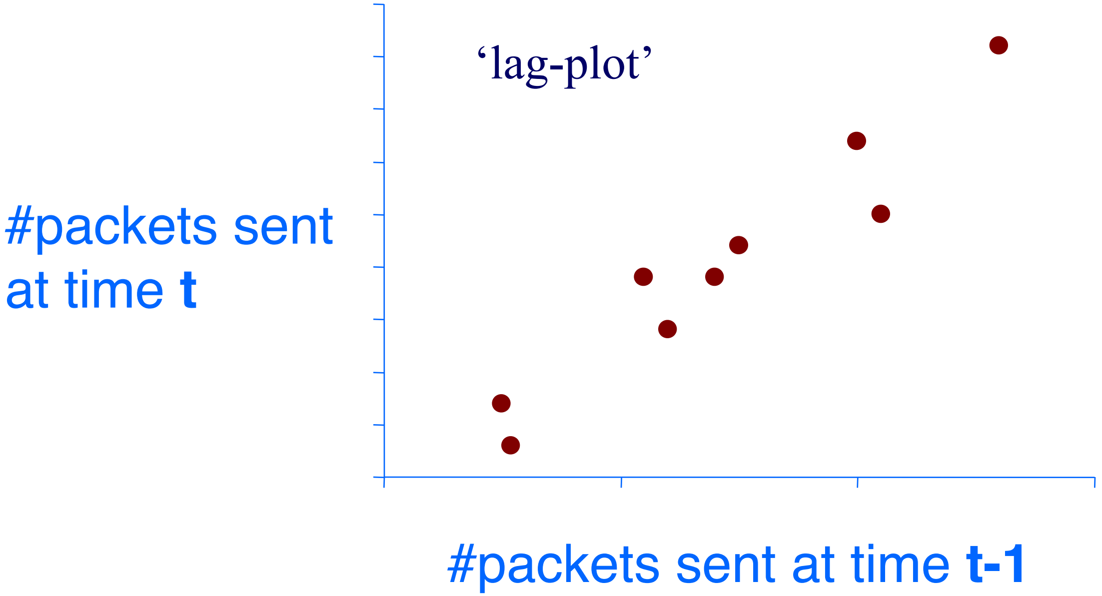
Express what we **don't know** (= "dependent variable")
as a linear function of what we **know** (= "independent variable(s)")

Linear **Auto** Regression

Time	*Packets Sent(t)*
1	43
2	54
3	72
…	…
N	**??**

Linear **Auto** Regression

Time	*Packets Sent (t-1)*	*Packets Sent(t)*
1	-	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



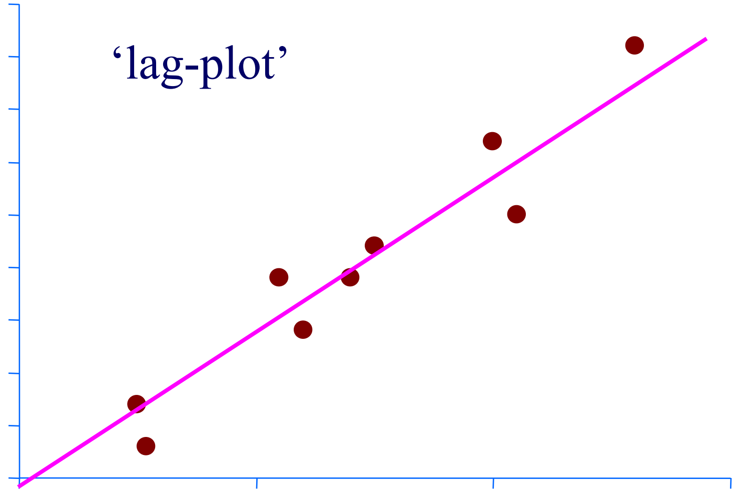
Lag *w* = 1

Dependent variable = # of packets sent (S [t])

Independent variable = # of packets sent (S[t-1])

Linear Auto Regression

Time	*Packets Sent (t-1)*	*Packets Sent(t)*
1	-	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



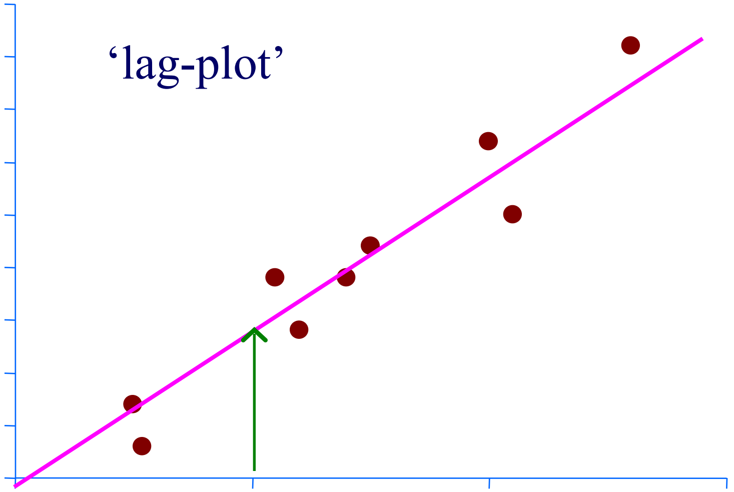
Lag *w* = 1

Dependent variable = # of packets sent (S [t])

Independent variable = # of packets sent (S[t-1])

Linear Auto Regression

Time	*Packets Sent (t-1)*	*Packets Sent(t)*
1	-	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



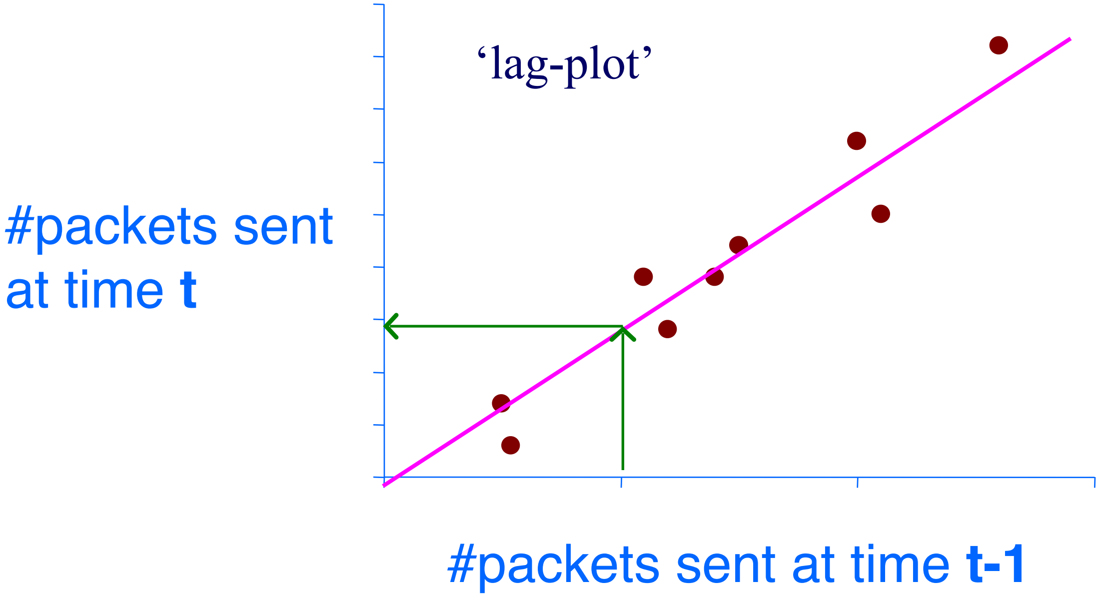
Lag $w = 1$

Dependent variable = # of packets sent (S [t])

Independent variable = # of packets sent (S[t-1])

Linear Auto Regression

Time	*Packets Sent (t-1)*	*Packets Sent(t)*
1	-	43
2	43	54
3	54	72
	…	
…		…
N	**25**	**??**



Lag *w* = 1

Dependent variable = # of packets sent (S [t])

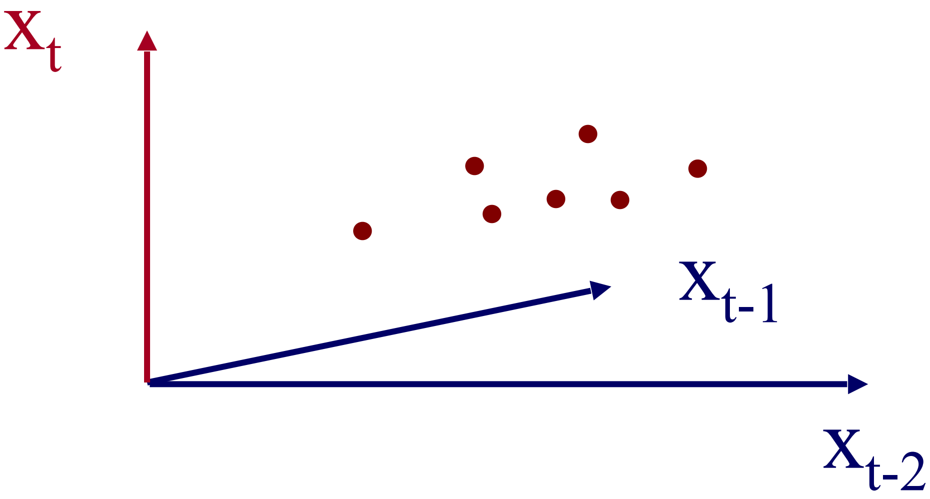
Independent variable = # of packets sent (S[t-1])

Outline

- Motivation
- ...
- Linear Forecasting
  - Auto-regression: **Least Squares; RLS**
  - Co-evolving time sequences
  - Examples
  - Conclusions

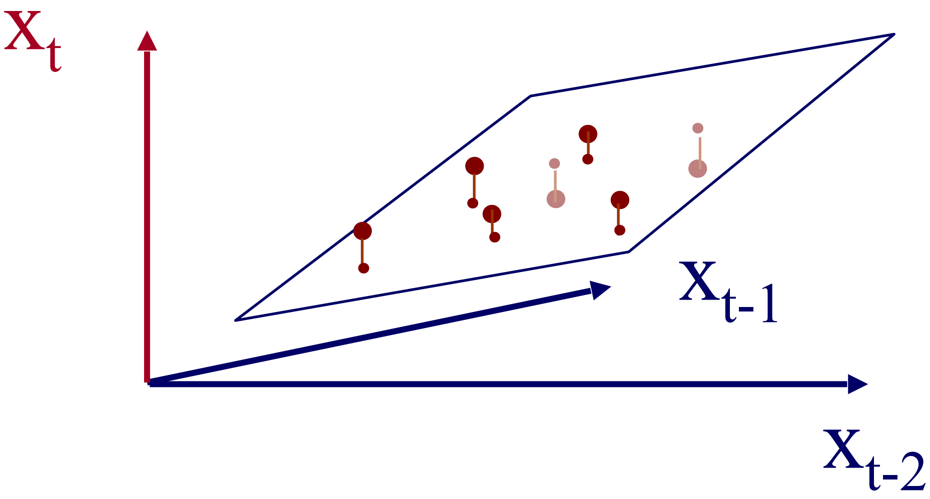
More details:

- Q1: Can it work with window $w > 1$?
- A1: YES!



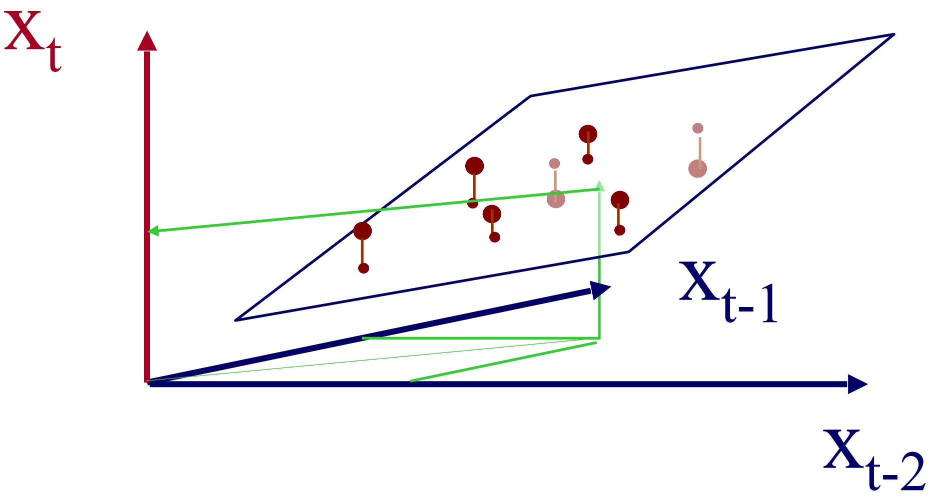
More details:

- Q1: Can it work with window *w* > 1?
- A1: YES! (we'll fit a hyper-plane, then!)



More details:

- Q1: Can it work with window $w > 1$?
- A1: YES! (we'll fit a hyper-plane, then!)



More details:

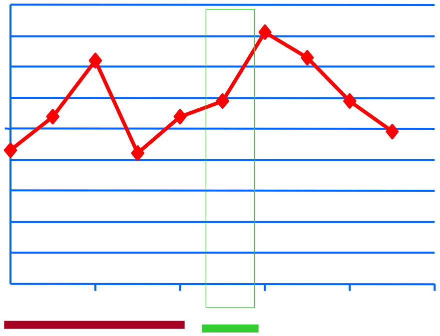
- Q1: Can it work with window $w > 1$?
- A1: YES! The problem becomes:

$$\mathbf{X}_{[N \times w]} \times \mathbf{a}_{[w \times 1]} = \mathbf{y}_{[N \times 1]}$$

- OVER-CONSTRAINED
  - **a** is the vector of the regression coefficients
  - **X** has the *N* values of the *w* indep. variables
  - **y** has the N values of the dependent variable

More details:

- $\mathbf{X}_{[N \times w]} \times \mathbf{a}_{[w \times 1]} = \mathbf{y}_{[N \times 1]}$

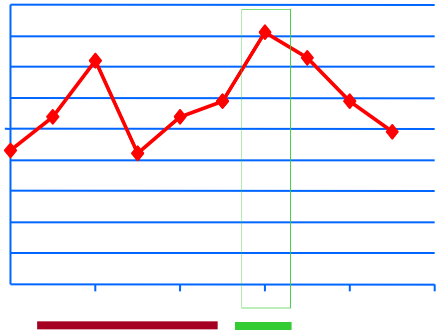


Ind-var1 Ind-var-w

time

$$\begin{bmatrix} \underline{X_{11}, X_{12}, \cdots, X_{1w}} \\ X_{21}, X_{22}, \ldots, X_{2w} \\ \vdots \\ \vdots \\ \vdots \\ X_{N1}, X_{N2}, \ldots, X_{Nw} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_w \end{bmatrix} = \begin{bmatrix} \underline{y_1} \\ y_2 \\ \vdots \\ \vdots \\ \vdots \\ y_N \end{bmatrix}$$

More details:



- $\mathbf{X}_{[N \times w]} \times \mathbf{a}_{[w \times 1]} = \mathbf{y}_{[N \times 1]}$

Ind-var1 Ind-var-w

time

$$\begin{bmatrix} X_{11}, X_{12}, \cdots, X_{1w} \\ \underline{X_{21}, X_{22}, \ldots, X_{2w}} \\ \vdots \\ \vdots \\ \vdots \\ X_{N1}, X_{N2}, \ldots, X_{Nw} \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_w \end{bmatrix} = \begin{bmatrix} y_1 \\ \underline{y_2} \\ \vdots \\ \vdots \\ \vdots \\ y_N \end{bmatrix}$$

More details

- Q2: How to estimate a_1, a_2, … a_w $= \mathbf{a}$?
- A2: with Least Squares fit

$$\boxed{\mathbf{a} = (\mathbf{X}^T \times \mathbf{X})^{-1} \times (\mathbf{X}^T \times \mathbf{y})}$$

- (Moore-Penrose pseudo-inverse)
- **a** is the vector that minimizes the RMSE from **y**

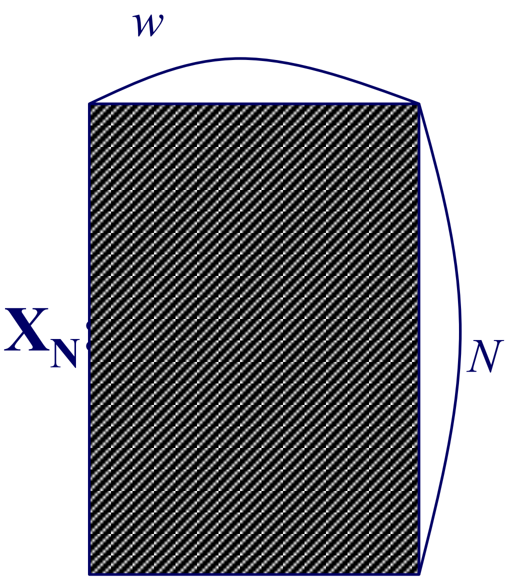
More details

- Straightforward solution:

$$\boxed{\mathbf{a} = (\mathbf{X}^T \times \mathbf{X})^{-1} \times (\mathbf{X}^T \times \mathbf{y})}$$

$\mathbf{a}$: Regression Coeff. Vector

$\mathbf{X}$: Sample Matrix



- Observations:
  - Sample matrix X grows over time
  - needs matrix inversion
  - $\mathbf{O}(N \times w^2)$ computation
  - $\mathbf{O}(N \times w)$ storage

Even more details

- Q3: Can we estimate **a** incrementally?
- A3: Yes, with the brilliant, classic method of “Recursive Least Squares” (RLS) (see, e.g., [Yi+00], for details).
- We can do the matrix inversion, WITHOUT inversion! (How is that possible?!)

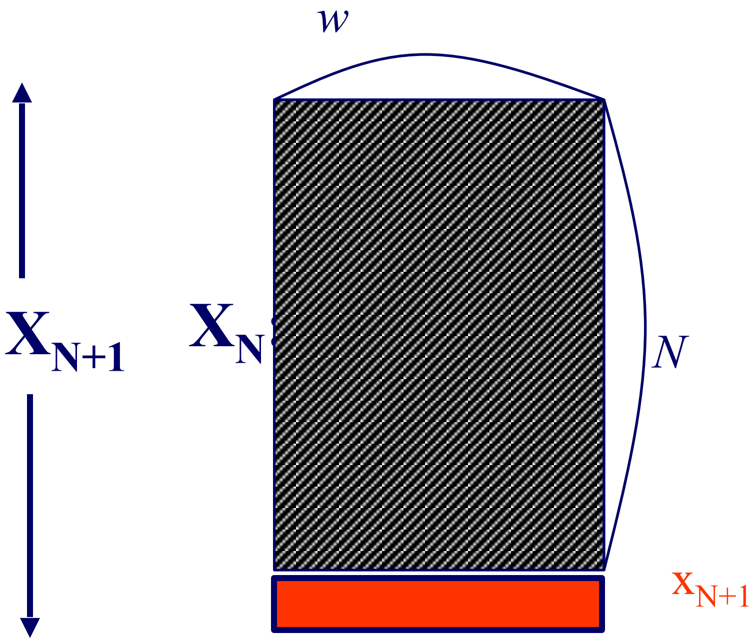
Even more details

- Q3: Can we estimate **a** incrementally?
- A3: Yes, with the brilliant, classic method of **"Recursive Least Squares" (RLS)** (see, e.g., [Yi+00], for details).
- We can do the matrix inversion, WITHOUT inversion! (How is that possible?!)
- A: our matrix has special form: $(X^T X)$

SKIP

More details

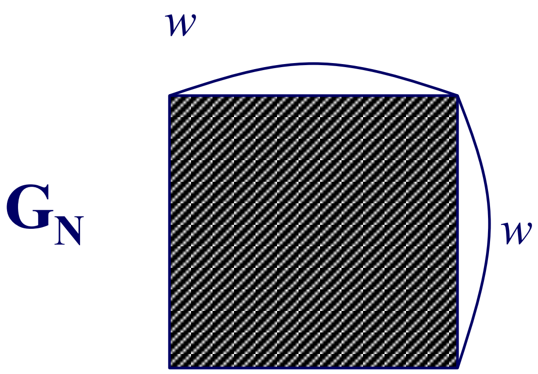
At the *N+1* time tick:



SKIP

More details: key ideas

- Let $\mathbf{G}_N = (\mathbf{X}_N^T \times \mathbf{X}_N)^{-1}$ (“gain matrix”)
- $\mathbf{G}_{N+1}$ can be computed recursively from $\mathbf{G}_N$ without matrix inversion



Comparison:

- Straightforward Least Squares
  - Needs huge matrix (**growing** in size) $O(N \times w)$
  - Costly matrix operation $O(N \times w^2)$

- Recursive LS
  - Need much smaller, fixed size matrix $O(w \times w)$
  - Fast, incremental computation $O(1 \times w^2)$
  - **no matrix inversion**

$$\boxed{N = 10^6, \quad w = 1\text{-}100}$$

SKIP

EVEN more details:

$$G_{N+1} = G_N - [c]^{-1} \times [G_N \times x_{N+1}{}^T] \times x_{N+1} \times G_N$$

1 x *w* row vector

$$c = [1 + x_{N+1} \times G_N \times x_{N+1}{}^T]$$

Let’s elaborate
(VERY IMPORTANT, VERY VALUABLE!)

SKIP

EVEN more details:

$$a = [X_{N+1}{}^T \times X_{N+1}]^{-1} \times [X_{N+1}{}^T \times y_{N+1}]$$

SKIP

EVEN more details:

$$a = [X_{N+1}{}^{T} \times X_{N+1}]^{-1} \times [X_{N+1}{}^{T} \times y_{N+1}]$$

[w x 1]   [w x (N+1)]   [(N+1) x w]   [w x (N+1)]   [(N+1) x 1]

SKIP

EVEN more details:

$$a = \boxed{[X_{N+1}^{\;T} \times X_{N+1}]^{\boxed{-1}}} \times [X_{N+1}^{\;T} \times y_{N+1}]$$

[w x (N+1)]　　　[(N+1) x w]

SKIP

EVEN more details:

$$a = [X_{N+1}^T \times X_{N+1}]^{-1} \times [X_{N+1}^T \times y_{N+1}]$$

‘gain matrix’ $$G_{N+1} \equiv [X_{N+1}^T \times X_{N+1}]^{-1}$$

$$\overset{\text{wxw}}{G_{N+1}} = \overset{\text{wxw}}{G_N} - \overset{\text{1x1}}{[c]^{-1}} \times [\overset{\text{wxw}}{G_N} \times \overset{\text{wx1}}{x_{N+1}^T}] \times \overset{\text{1xw}}{x_{N+1}} \times \overset{\text{wxw}}{G_N}$$

SCALAR! $$c = [1 + x_{N+1} \times G_N \times x_{N+1}^T]$$

SKIP

Altogether:

$$G_0 \equiv \delta\, I$$

where
I: w x w identity matrix
δ: a large positive number

Comparison:

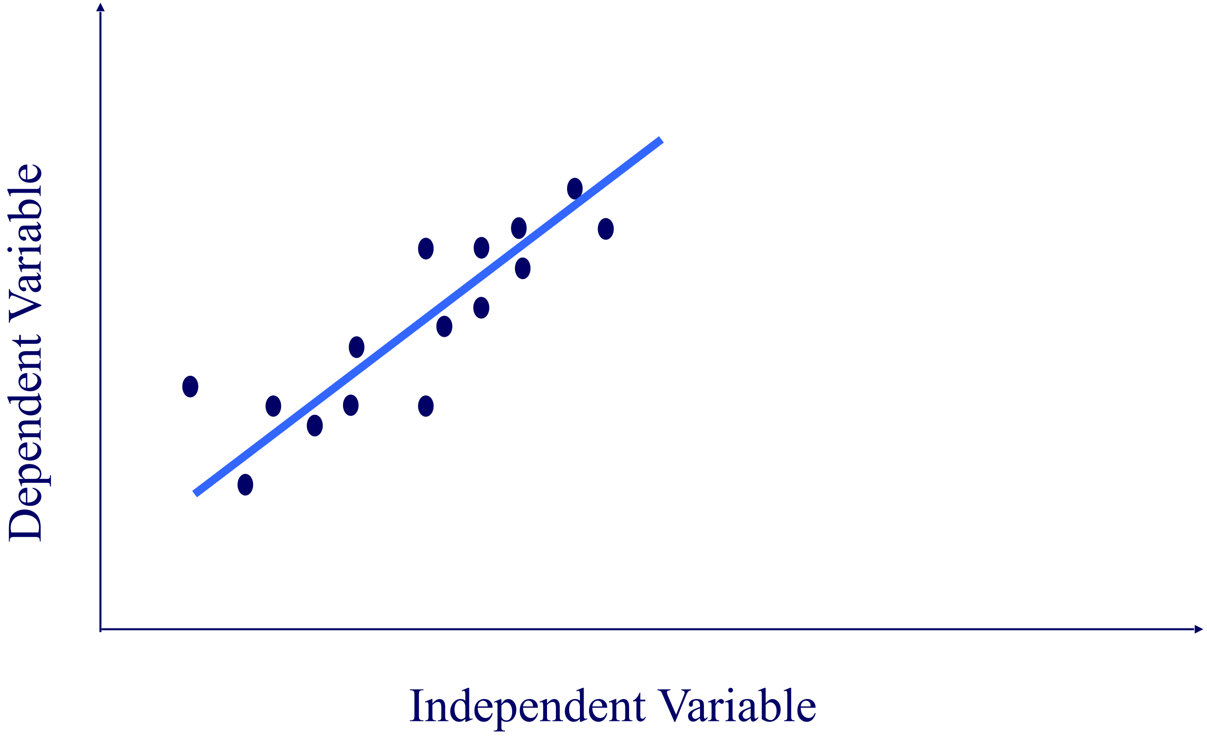
- Straightforward Least Squares
  - Needs huge matrix (**growing** in size) $O(N \times w)$
  - Costly matrix operation $O(N \times w^2)$

- Recursive LS
  - Need much smaller, fixed size matrix $O(w \times w)$
  - Fast, incremental computation $O(1 \times w^2)$
  - **no matrix inversion**

$$\boxed{N = 10^6, \quad w = 1\text{-}100}$$

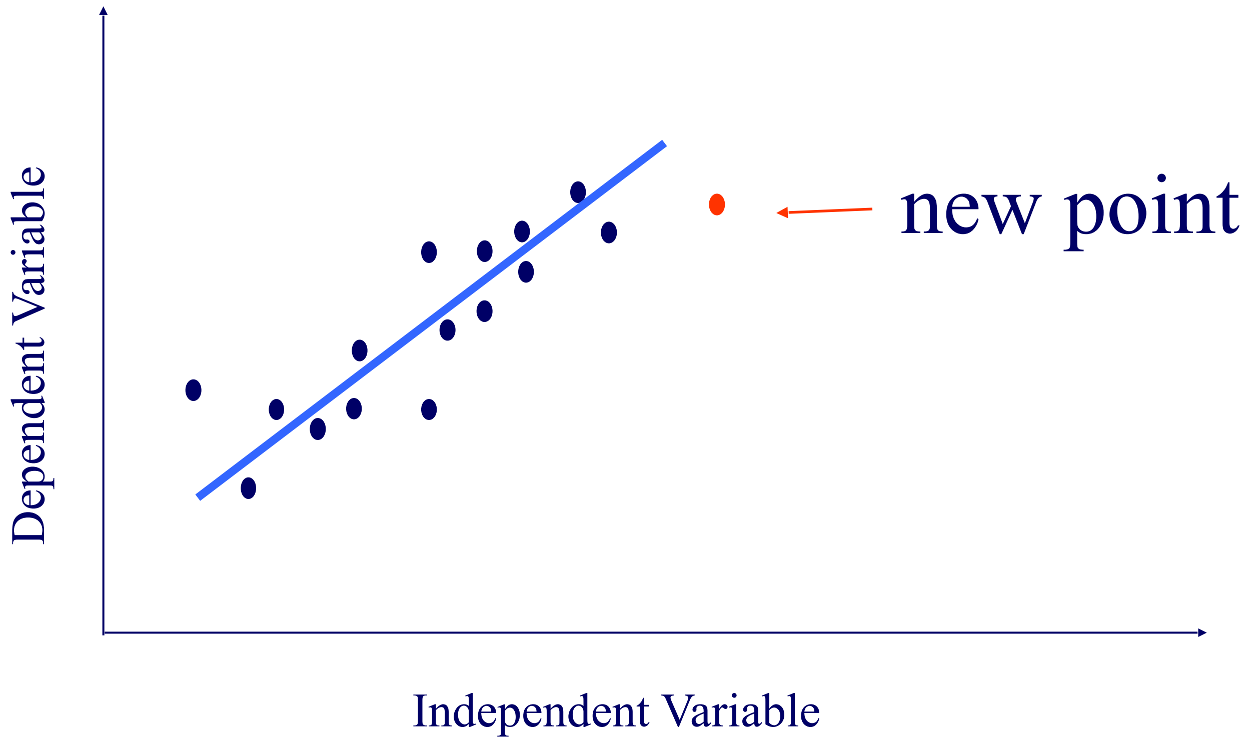
Pictorially:

- Given:



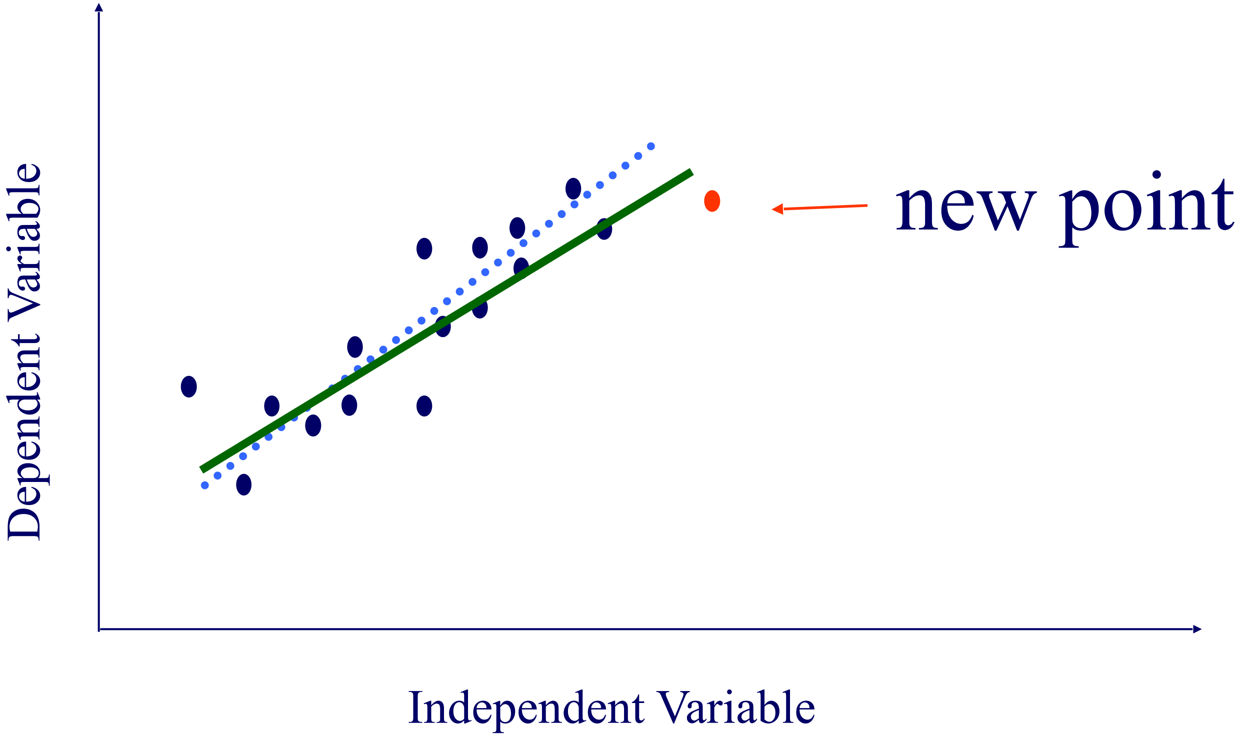
Pictorially:

-



Pictorially:

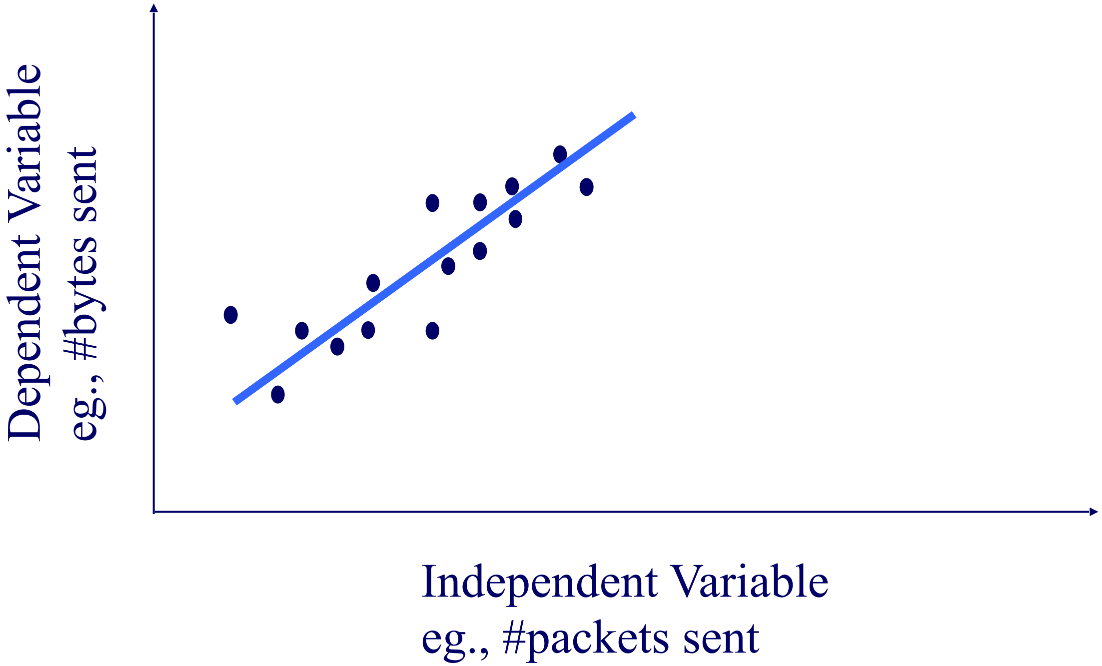
RLS: quickly compute new best fit



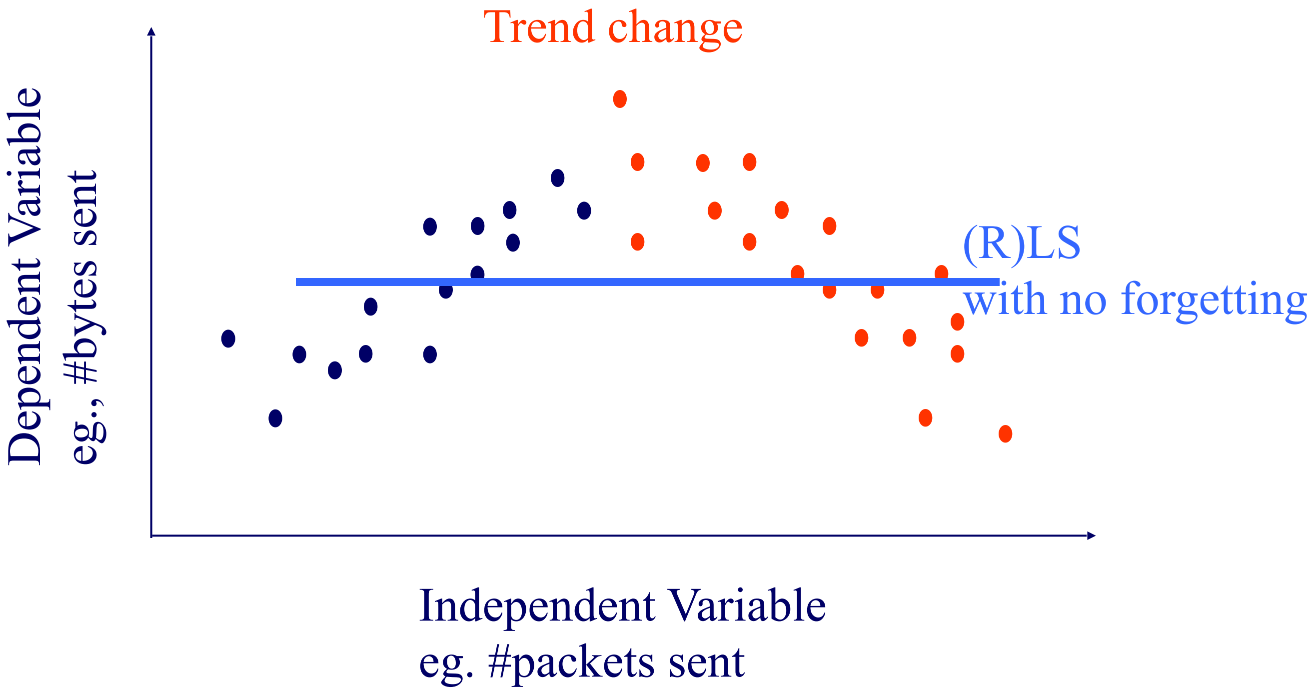
Even more details

- Q4: can we ‘forget’ the older samples?
- A4: Yes - RLS can easily handle that [Yi+00]:

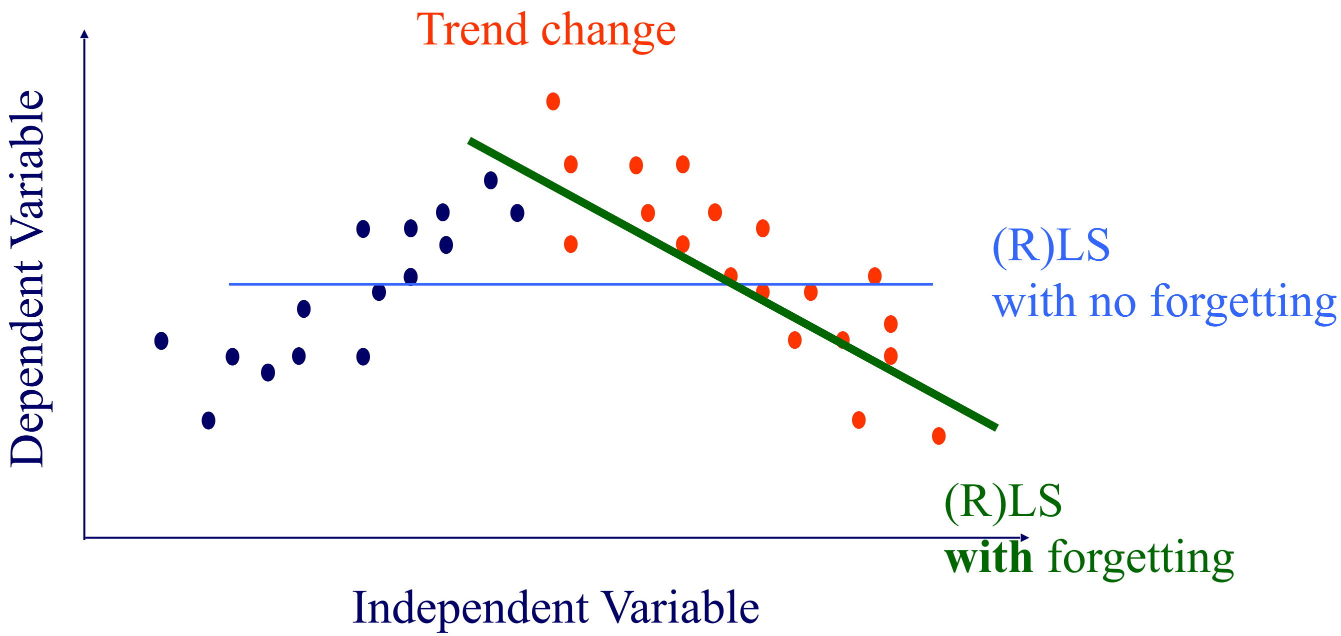
Adaptability - ‘forgetting’



Adaptability - ‘forgetting’



Adaptability - ‘forgetting’



- RLS: can *trivially* handle ‘forgetting’