http://poloclub.gatech.edu/cse6242

CSE6242 / CX4242: Data & Visual Analytics

Graphs / Networks

Centrality measures, algorithms, interactive applications

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Partly based on materials by
Professors Guy Lebanon, Jeffrey Heer, John Stasko, Christos Faloutsos, Parishit Ram (GT PhD alum; SkyTree), Alex Gray

Centrality

= “Importance”

Why Node Centrality?

What can we do if we can rank all the nodes in a graph (e.g., Facebook, LinkedIn, Twitter)?

- Find **celebrities** or influential people in a social network (Twitter)
- Find "**gatekeepers**" who connect communities (headhunters love to find them on LinkedIn)
- What else?



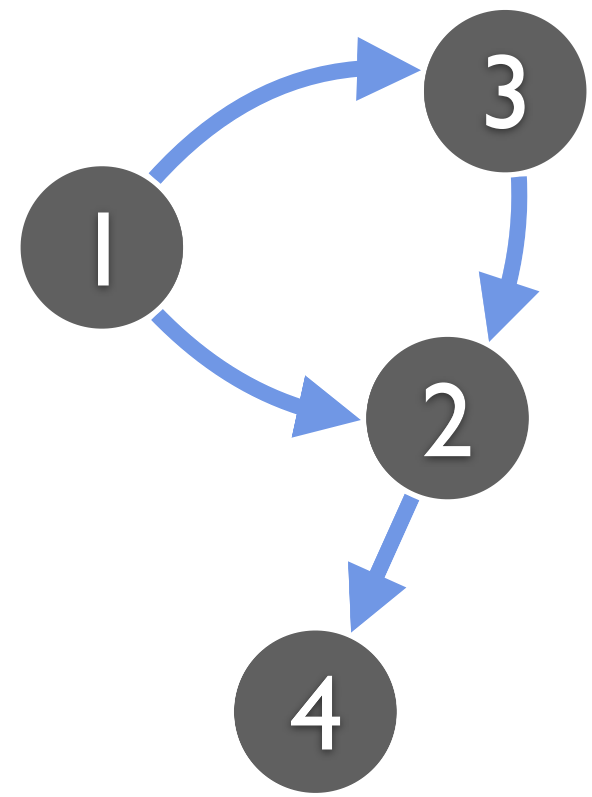
More generally

Helps **graph analysis, visualization, understanding**, e.g.,

- Let us **rank** nodes, group or study them by centrality
- Only show subgraph formed by the **top 100 nodes**, out of the millions in the full graph
  - **Similar to google search results** (ranked, and they only show you 10 per page)
- Most graph analysis packages already have centrality algorithms implemented. **Use them!**

Can also compute edge centrality.
Here we focus on node centrality.

Degree Centrality (easiest)



Degree = number of neighbors

- For directed graphs
  - **In degree** = No. of incoming edges
  - **Out degree** = No. of outgoing edges
- For undirected graphs, **only degree is defined**.
- Algorithms?
  - Sequential scan through edge list
  - What about for a graph stored in SQLite?

Computing Degrees using SQL

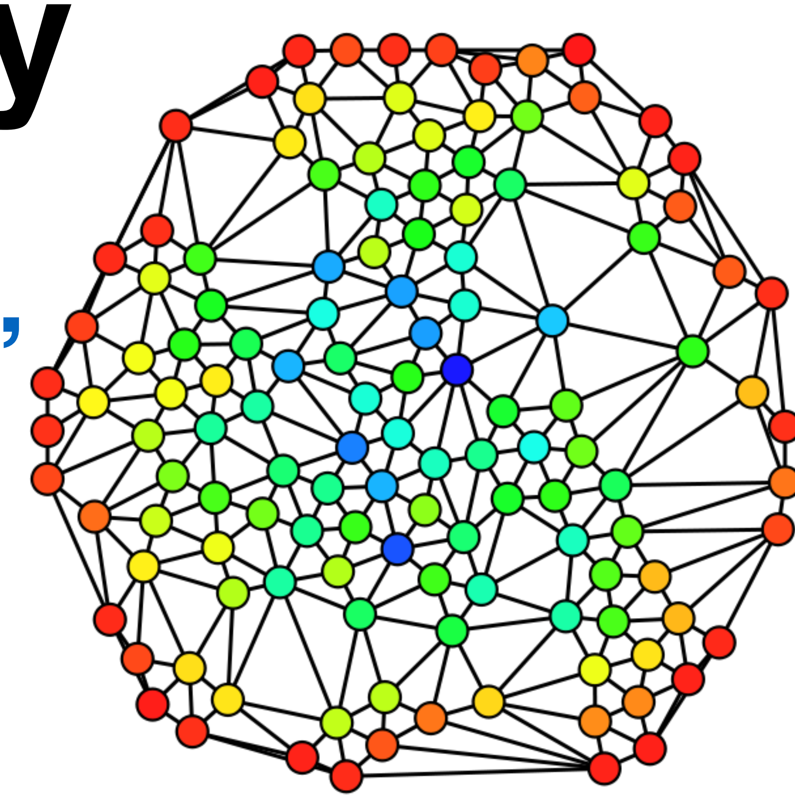
Recall simplest way to store a graph in SQLite:

```
edges(source_id, target_id)
```

1. If slow, first create index for each column
2. Use **group by** statement to find **in degrees**

```
select count(*) from edges group by source_id;
```

Betweenness Centrality



High betweenness = "gatekeeper"

Betweenness of a node v

$$= \sum_{s \neq v \neq t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$$

Number of shortest paths between s and t that **goes through v**

Number of shortest paths between s and t

= how often a node serves as the "bridge" that connects two other nodes.

Betweenness is very well studied. http://en.wikipedia.org/wiki/Centrality#Betweenness_centrality

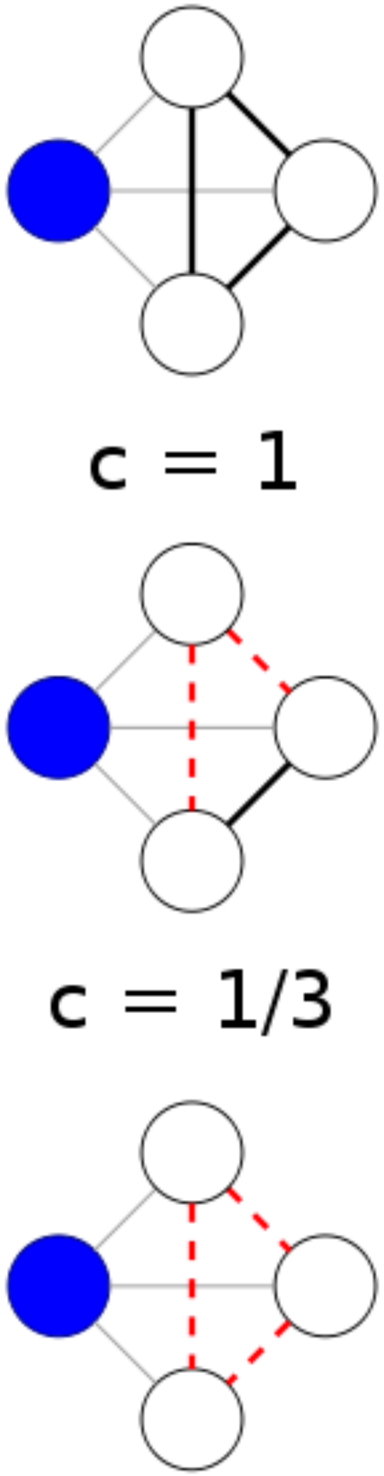
(Local) Clustering Coefficient

A node's clustering coefficient is a measure of how close the node's neighbors are from forming a clique.

- 1 = neighbors form a clique
- 0 = No edges among neighbors

(Assuming undirected graph)

"Local" means it's for a node; can also compute a graph's "global" coefficient



c = 1

c = 1/3

c = 0

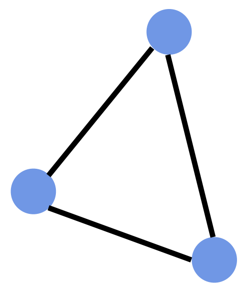
Image source: http://en.wikipedia.org/wiki/Clustering_coefficient

Computing Clustering Coefficients...

Requires **triangle counting**

Real social networks have a lot of triangles

- Friends of friends are friends



Triangles are **expensive** to compute

(neighborhood intersections; several approx. algos)

Can we do that quickly?

Algorithm details:
Faster Clustering Coefficient Using Vertex Covers
http://www.cc.gatech.edu/~ogreen3/_docs/2013VertexCoverClusteringCoefficients.pdf

details

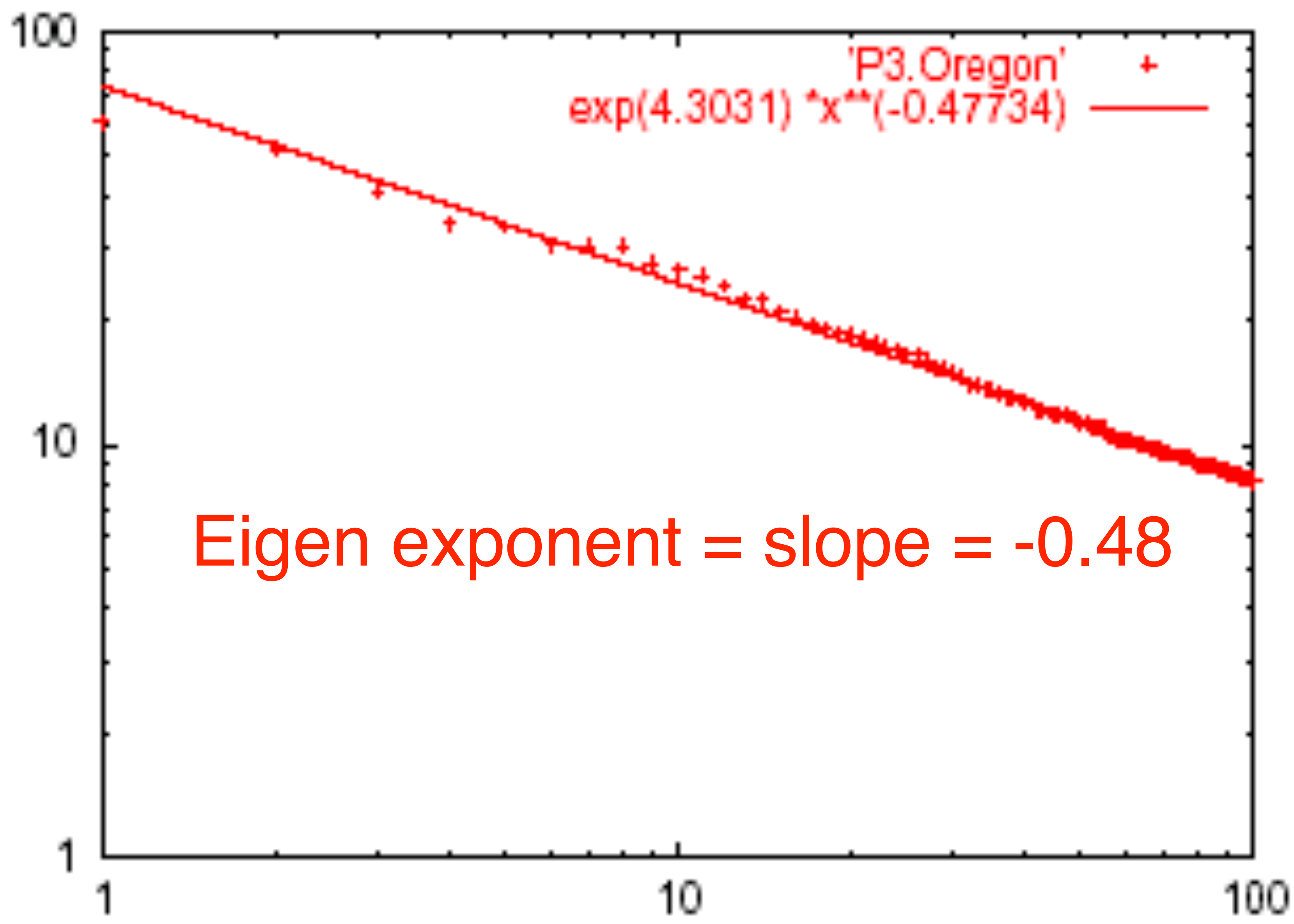
Super Fast Triangle Counting [Tsourakakis ICDM 2008]

But: triangles are expensive to compute
(3-way join; several approx. algos)
Q: Can we do that quickly?
A: Yes!

#triangles = 1/6 Sum (λ_i^3)

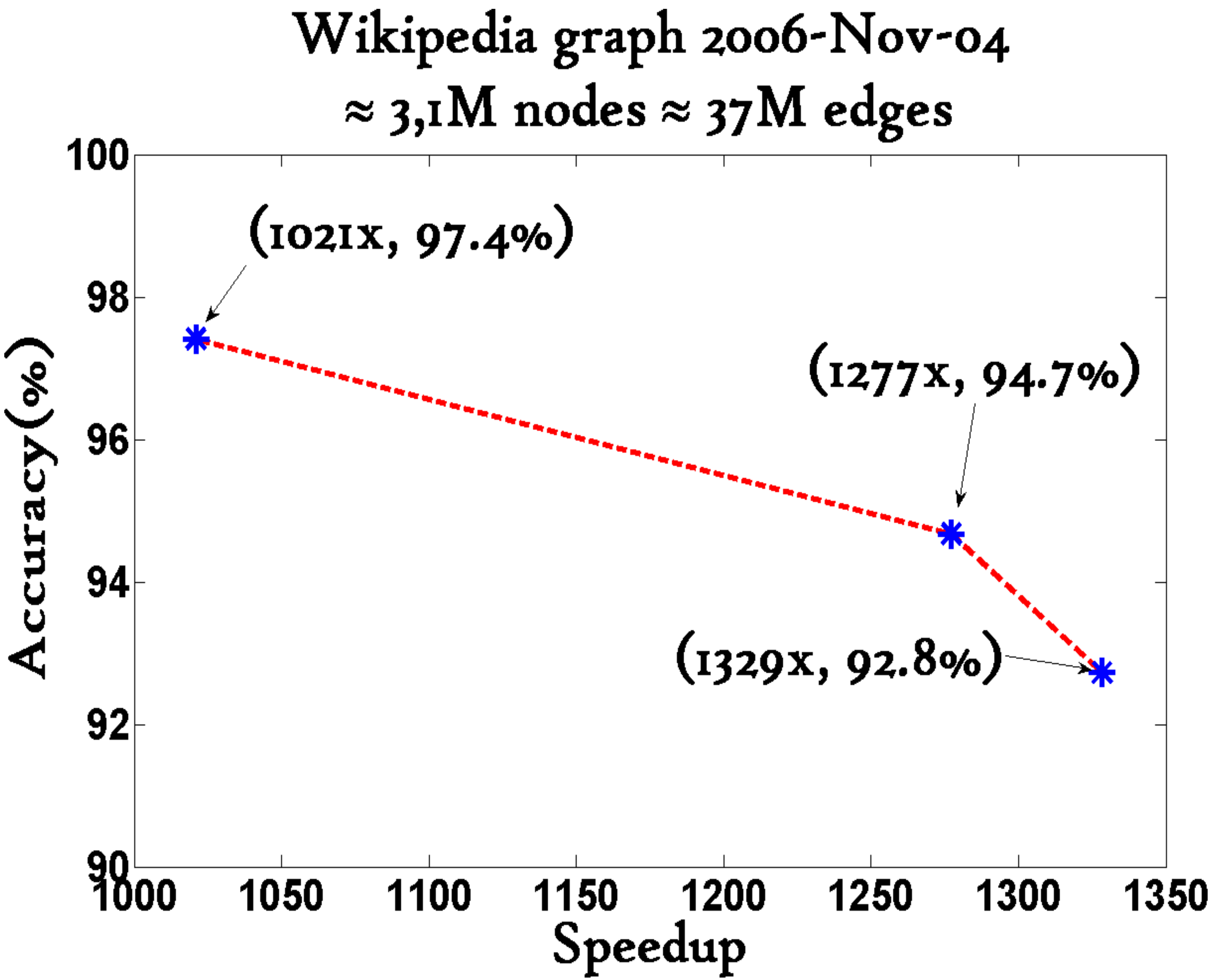
(and, because of skewness,
we only need the top few eigenvalues!

Power Law in Eigenvalues of Adjacency Matrix



Eigenvalue

Rank of decreasing eigenvalue



1000x+ speed-up, >90% accuracy

More Centrality Measures…

- Degree
- Betweenness
- Closeness, by computing
  - Shortest paths
  - "**Proximity**" (usually via *random walks*) — **used successfully in a lot of applications**
- Eigenvector
- …

PageRank (Google)

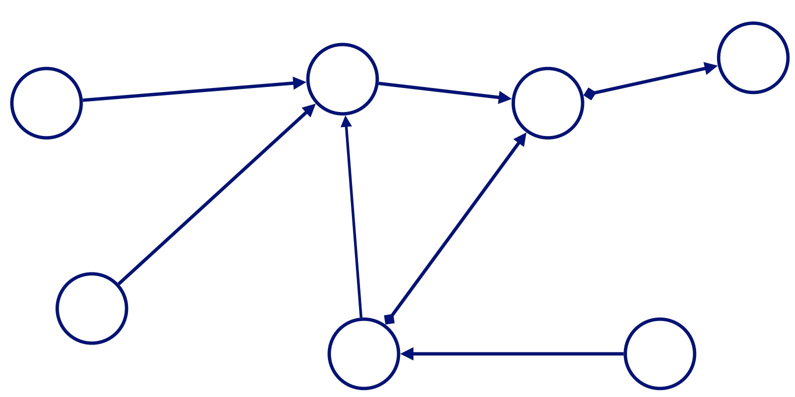
Larry Page

Sergey Brin

Brin, Sergey and Lawrence Page (1998). *Anatomy of a Large-Scale Hypertextual Web Search Engine*. 7th Intl World Wide Web Conf.

PageRank: Problem

Given a directed graph, find its most interesting/central node

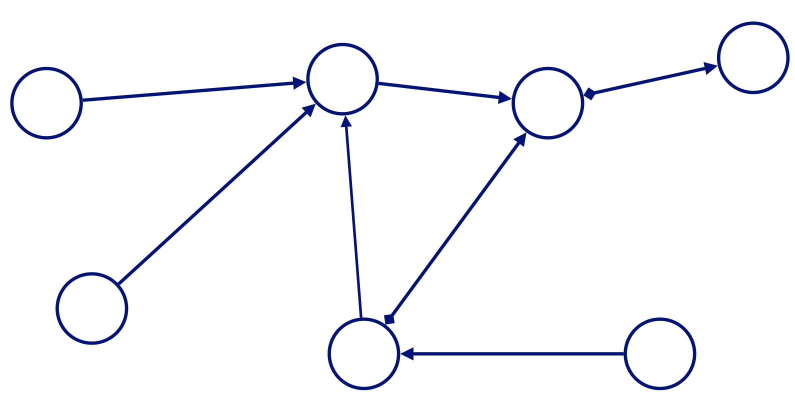


A node is important, if it is connected with important nodes (recursive, but OK!)

PageRank: Solution

Given a directed graph, find its most interesting/central node

Proposed solution:
use **random walk**; spot most “popular” node (-> steady state probability (ssp))

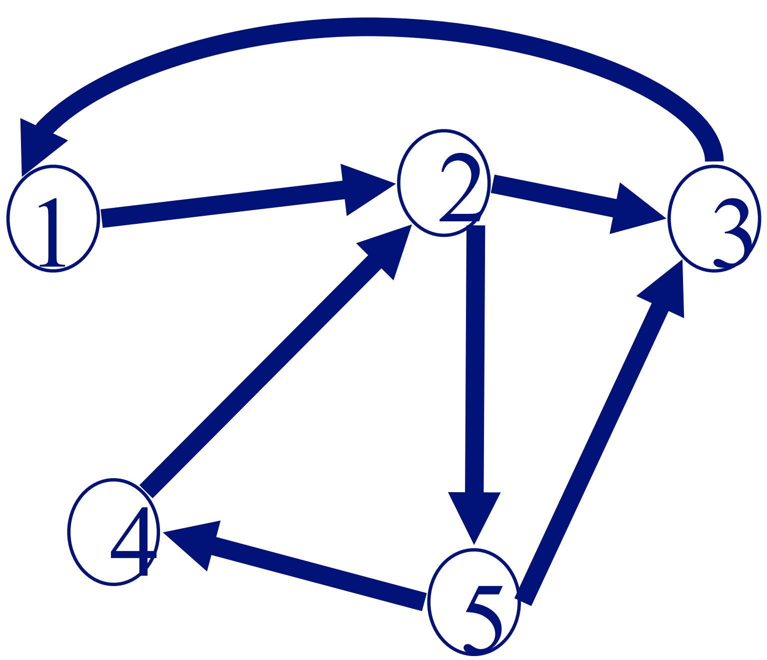


“state” = webpage

A node has high ssp,
if it is connected
with high ssp nodes
(recursive, but OK!)

(Simplified) PageRank

Let **B** be the transition matrix:
transposed, column-normalized



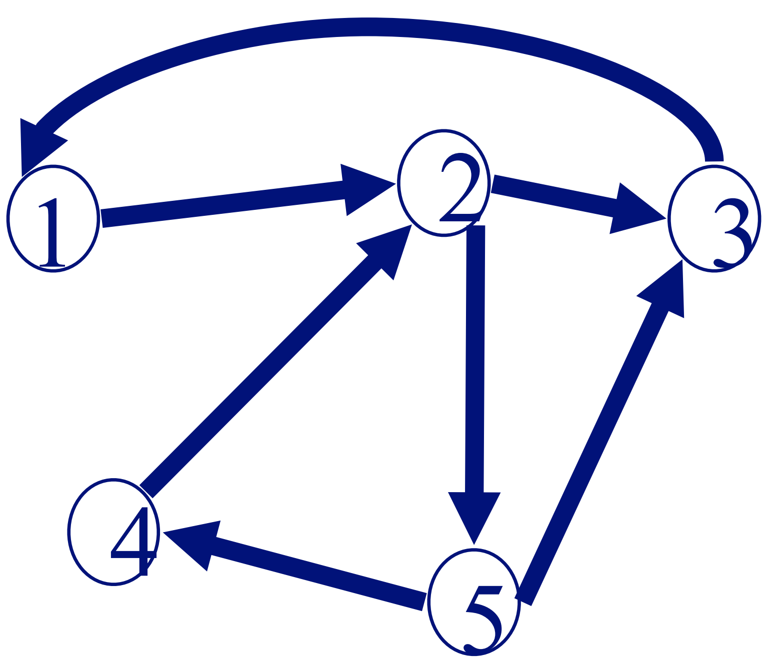
From **B**

To

$$\begin{bmatrix} & & 1 & & \\ 1 & & & 1 & \\ & 1/2 & & & 1/2 \\ & & & & 1/2 \\ & 1/2 & & & \end{bmatrix} \begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix} = \begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}$$

(Simplified) PageRank

B p = p



$$\underbrace{\begin{bmatrix} & & 1 & & \\ 1 & & & 1 & \\ & 1/2 & & & 1/2 \\ & & & & 1/2 \\ & 1/2 & & & \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}_{\mathbf{p}} = \underbrace{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}_{\mathbf{p}}$$

How to compute SSP:
https://fenix.tecnico.ulisboa.pt/downloadFile/3779579688473/6.3.pdf
http://www.sosmath.com/matrix/markov/markov.html

(Simplified) PageRank

- $\mathbf{B}\,\mathbf{p} = 1 * \mathbf{p}$
- Thus, $\mathbf{p}$ is the **eigenvector** that corresponds to the highest eigenvalue (=1, since the matrix is column-normalized)
- Why does such a $\mathbf{p}$ exist?
  - $\mathbf{p}$ exists if $\mathbf{B}$ is nxn, nonnegative, irreducible [Perron–Frobenius theorem]

(Simplified) PageRank

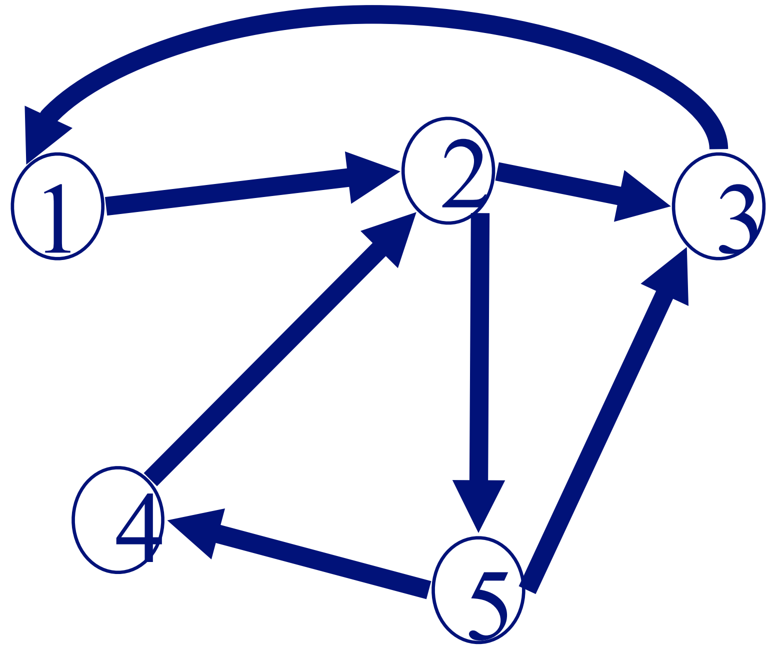
- In short: imagine a particle randomly moving along the edges
- Compute its steady-state probability (ssp)

Full version of algorithm:
with occasional random jumps

Why? To make the matrix irreducible

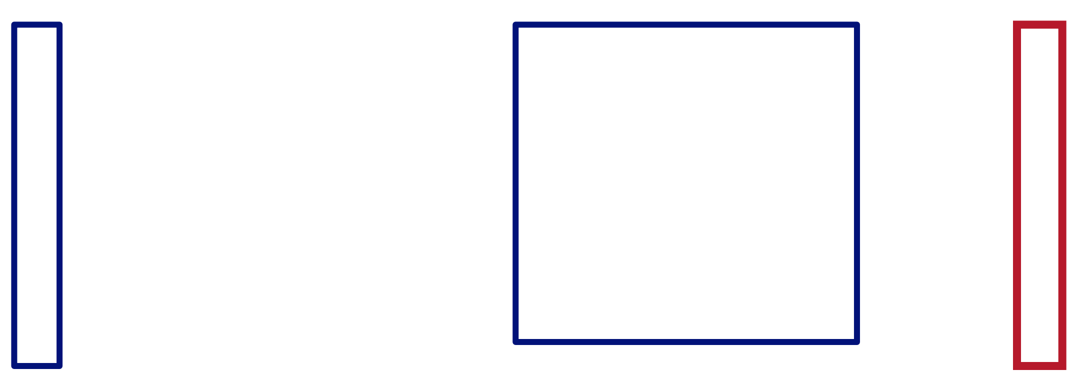
Full Algorithm

- With probability $1\text{-}c$, fly-out to a random node
- Then, we have



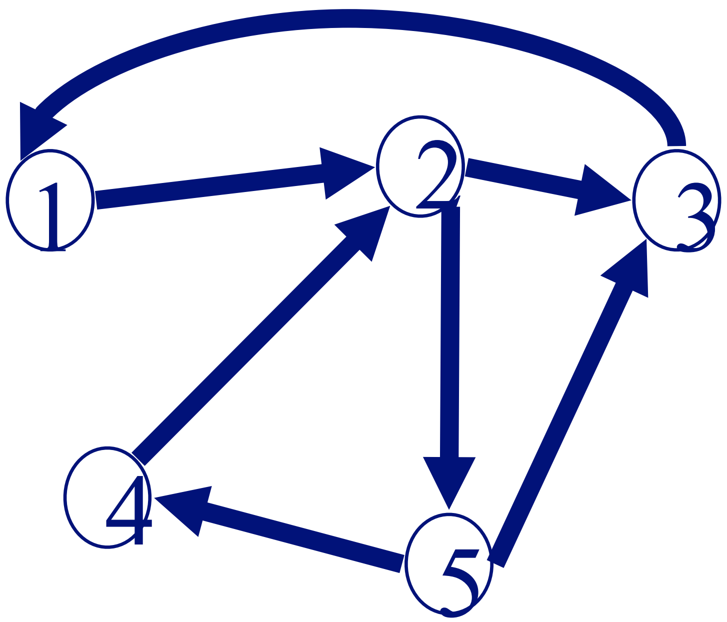
$$\mathbf{p} = c\ \mathbf{B}\ \mathbf{p} + (1\text{-}c)/n\ \mathbf{1} \Rightarrow$$

$$\mathbf{p} = (1\text{-}c)/n\ [\mathbf{I} - c\ \mathbf{B}]^{-1}\ \mathbf{1}$$



$$\begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}$$

How to compute PageRank for huge matrix?



Use the power iteration method

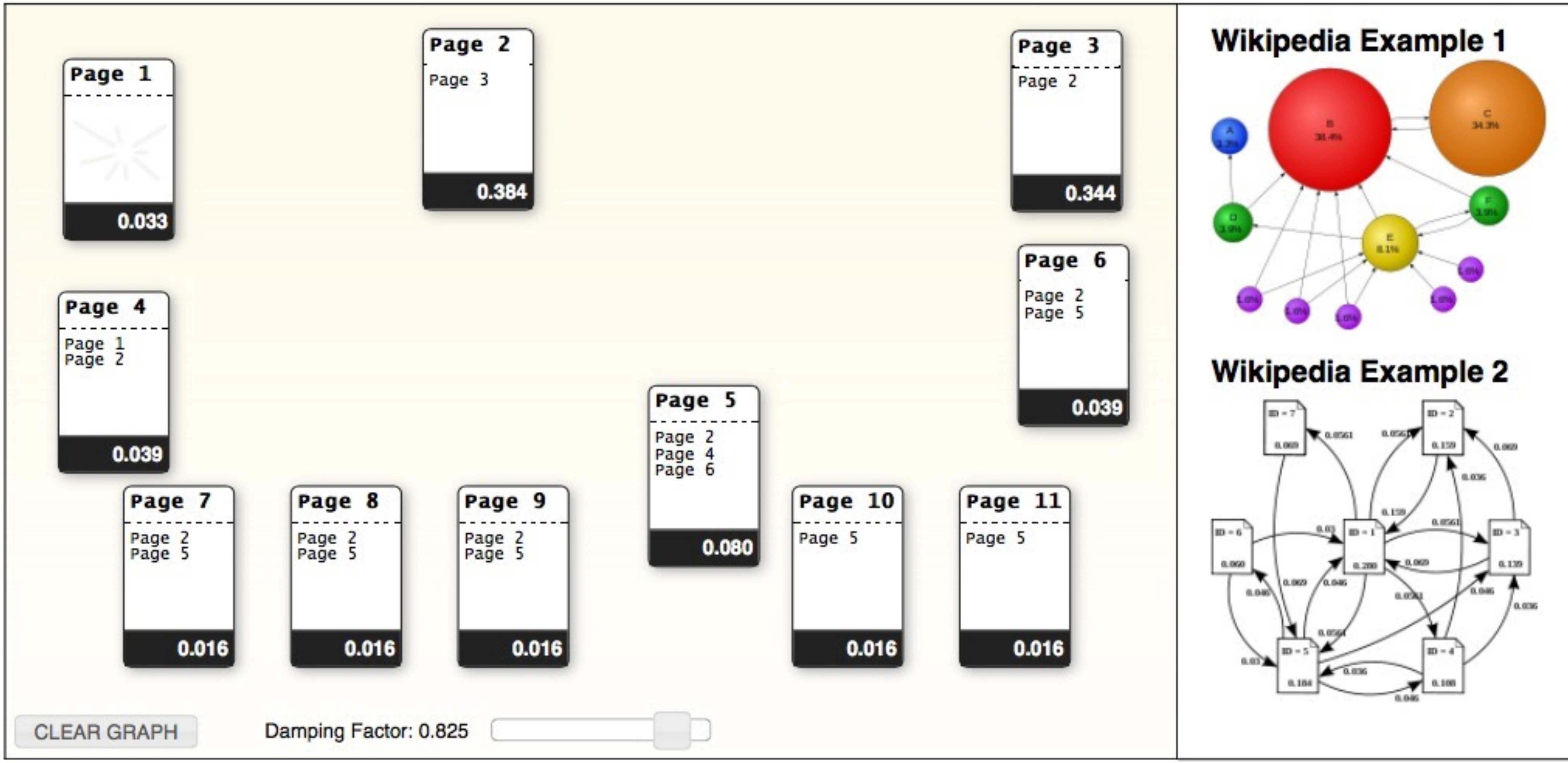
http://en.wikipedia.org/wiki/Power_iteration

$$\mathbf{p} = c\, \mathbf{B}\, \mathbf{p} + (1-c)/n\ \mathbf{1}$$

$$\underbrace{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}_{\mathbf{p'}} = c \underbrace{\begin{bmatrix} & & 1 & & \\ 1 & & & 1 & \\ & 1/2 & & & 1/2 \\ & & & & 1/2 \\ & 1/2 & & & \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}_{\mathbf{p}} + (1-c)1/n \begin{bmatrix} 1 \\ 1 \\ \dots \\ 1 \end{bmatrix}$$

Can initialize this vector to any non-zero vector, e.g., all “1”s

PageRank Explained with Javascript



http://www.cs.duke.edu/csed/principles/pagerank/

PageRank for graphs (generally)

You can compute PageRank for **any graphs**

Should be in your algorithm “toolbox”

- Better than simple centrality measure (e.g., degree)
- Fast to compute for large graphs (O(E))

But can be “misled” (Google Bomb)

- How?

Personalized PageRank

Make one small variation of PageRank

- Intuition: not all pages are equal, some more relevant to a person’s specific needs
- How?

Personalized PageRank

With probability $1\text{-}c$, fly-out to ~~a random node~~ **some preferred nodes**

$$\underbrace{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}_{\mathbf{p'}} = \mathbf{c} \underbrace{\begin{bmatrix} & & 1 & & \\ 1 & & & 1 & \\ & 1/2 & & & 1/2 \\ & & & & 1/2 \\ & 1/2 & & & \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} p1 \\ p2 \\ p3 \\ p4 \\ p5 \end{bmatrix}}_{\mathbf{p}} + (1\text{-}c)1/n \begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ \mathbf{1} \\ 0 \\ \cdots \\ 0 \\ \mathbf{1} \end{bmatrix}$$

Can initialize this vector to any non-zero vector, e.g., all “1”s

Why learn Personalized PageRank?

For **recommendation**

- If I like webpage A, what else do I like?
- If I bought product A, what other products would I also buy?

Visualizing and interacting with large graphs

- Instead of visualizing every single nodes, visualize the **most important ones**

Very flexible — works on any graph

Related "guilt-by-association" / diffusion techniques

- **Personalized PageRank**
  (= Random Walk with Restart)
- "Spreading activation" or "degree of interest" in Human-Computer Interaction (HCI)
- Belief Propagation
  (powerful inference algorithm, for fraud detection, image segmentation, error-correcting codes, etc.)

Why are these algorithms popular?

- **Intuitive to interpret**
  uses “network effect”, homophily
- **Easy to implement**
  Math is relatively simple (mainly matrix-vector multiplication)
- **Fast**
  run time linear to #edges, or better
- **Probabilistic** meaning

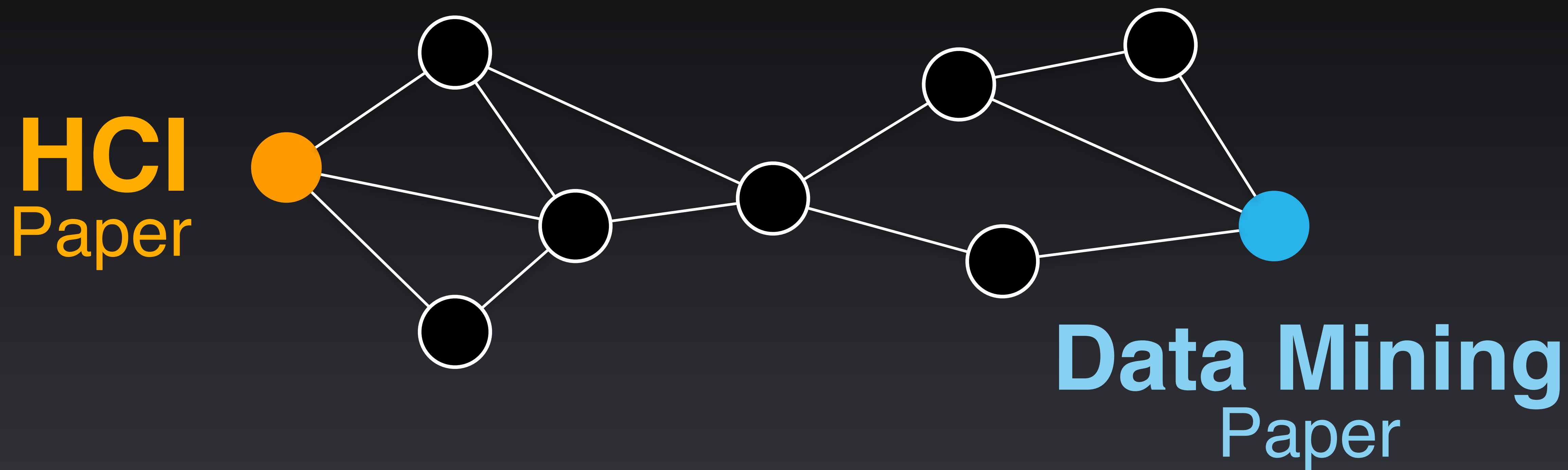
Human-In-The-Loop Graph Mining

Apolo:
Machine Learning + Visualization

CHI 2011

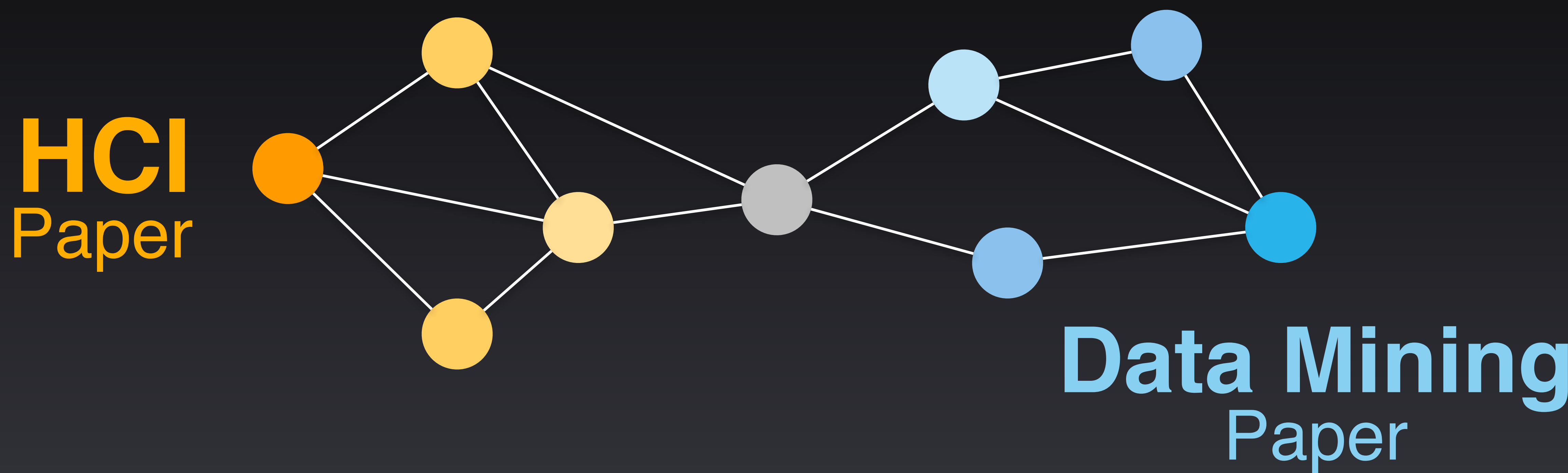
Apolo: Making Sense of Large Network Data by Combining Rich User Interaction and Machine Learning

Finding More Relevant Nodes



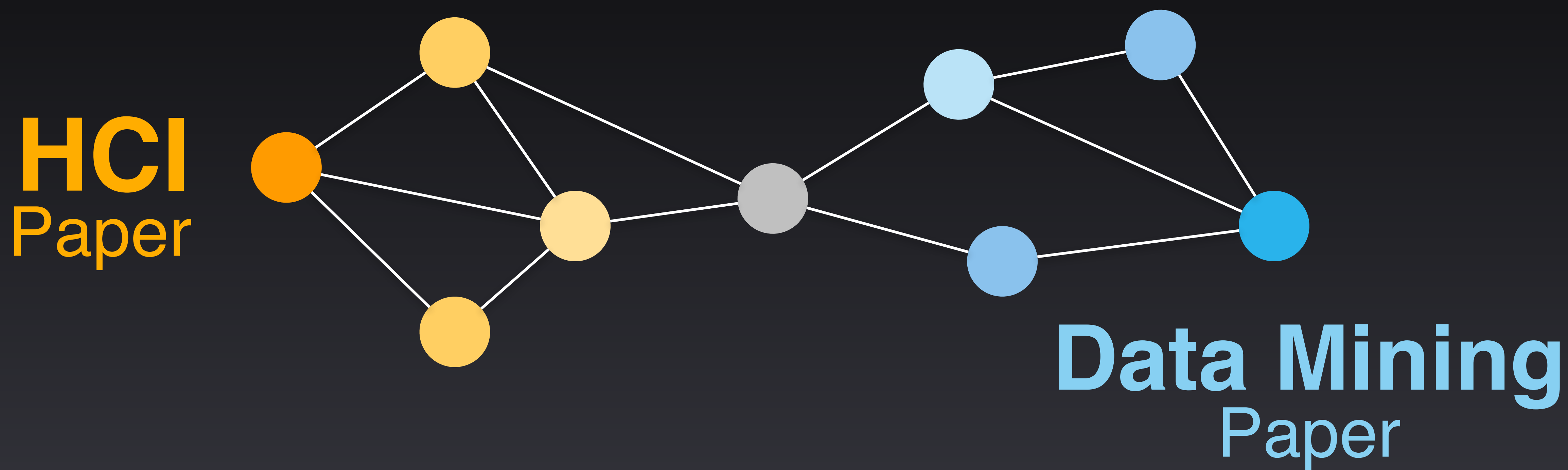
Citation network

Finding More Relevant Nodes



Citation network

Finding More Relevant Nodes



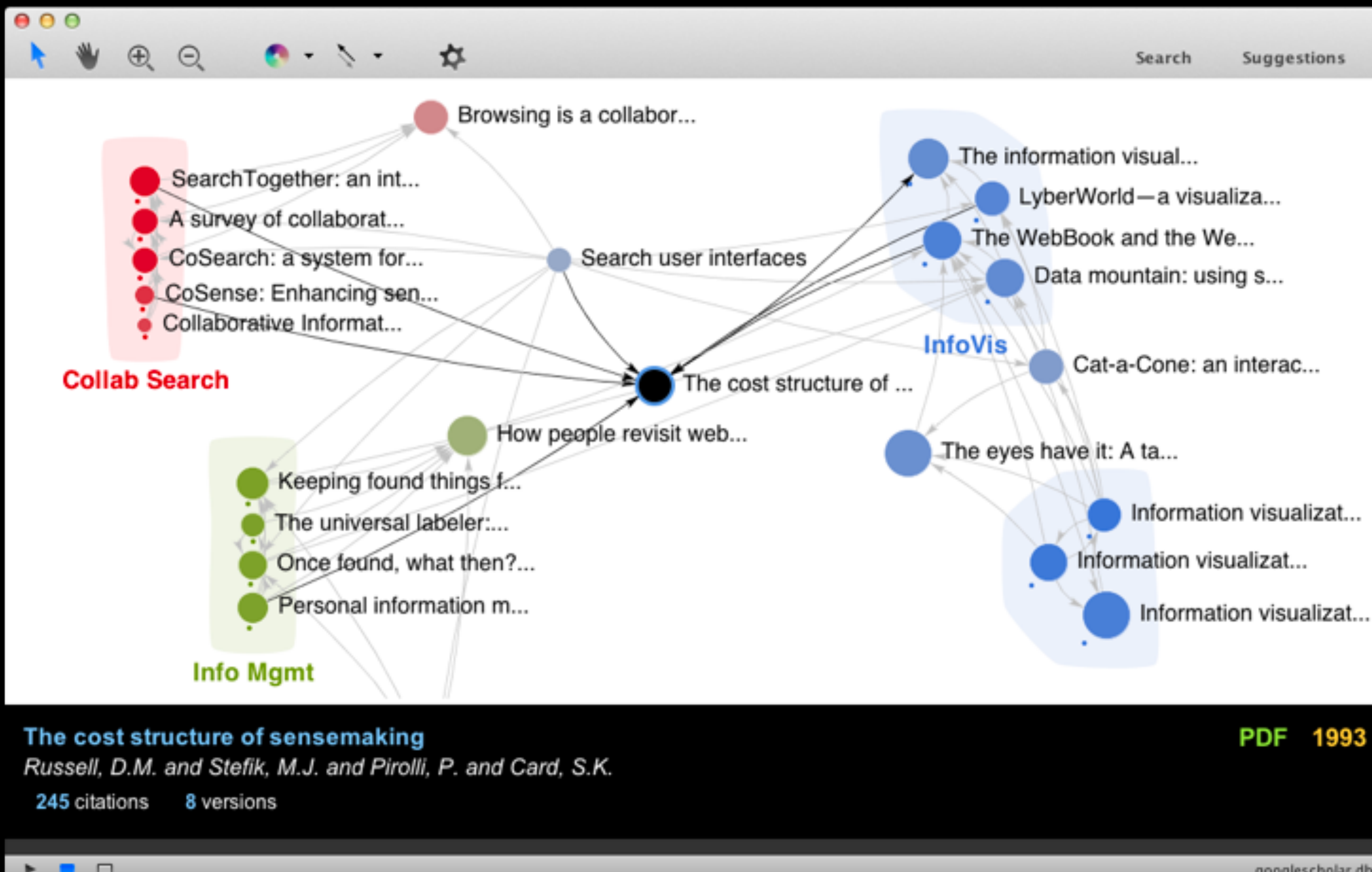
Citation network

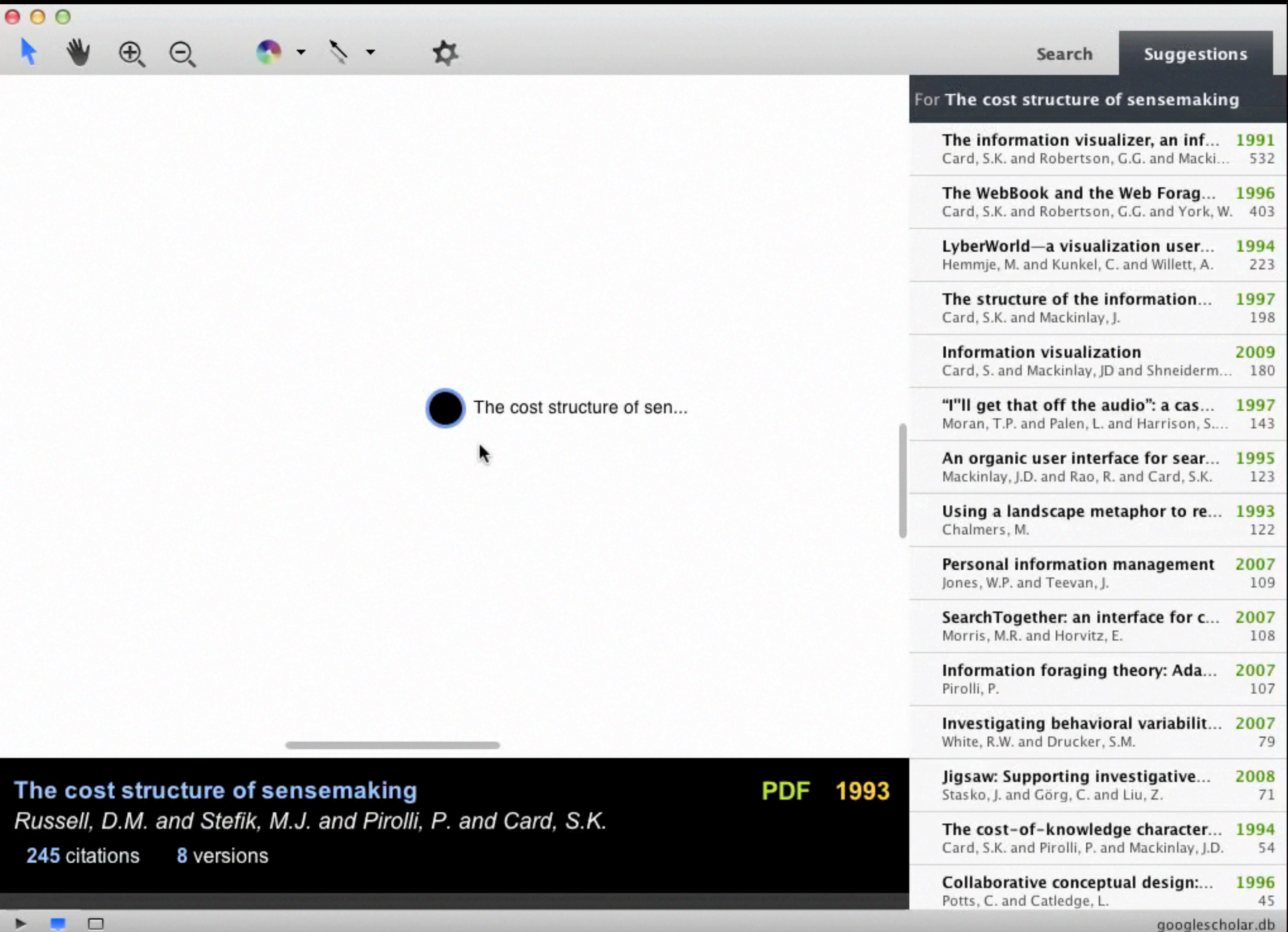
Apolo uses **guilt-by-association**

(Belief Propagation, similar to personalized PageRank)

Demo: Mapping the Sensemaking Literature

Nodes: 80k papers from Google Scholar (node size: #citation)
Edges: 150k citations

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The information visualizer, an inf... 1991
Card, S.K. and Robertson, G.G. and Macki... 532
The WebBook and the Web Forag... 1996
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PDF 1993
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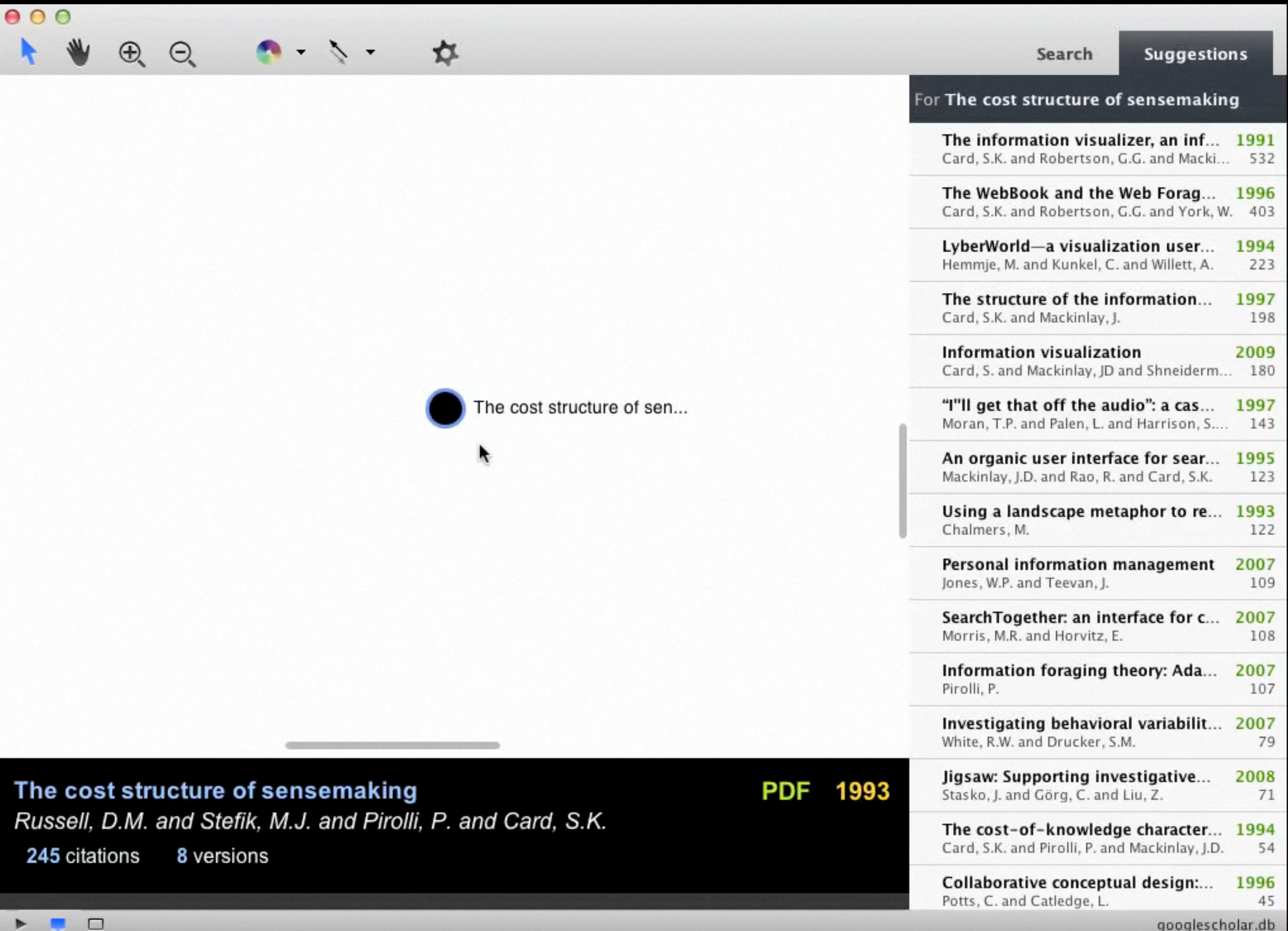
245 citations 8 versions
googlescholar.db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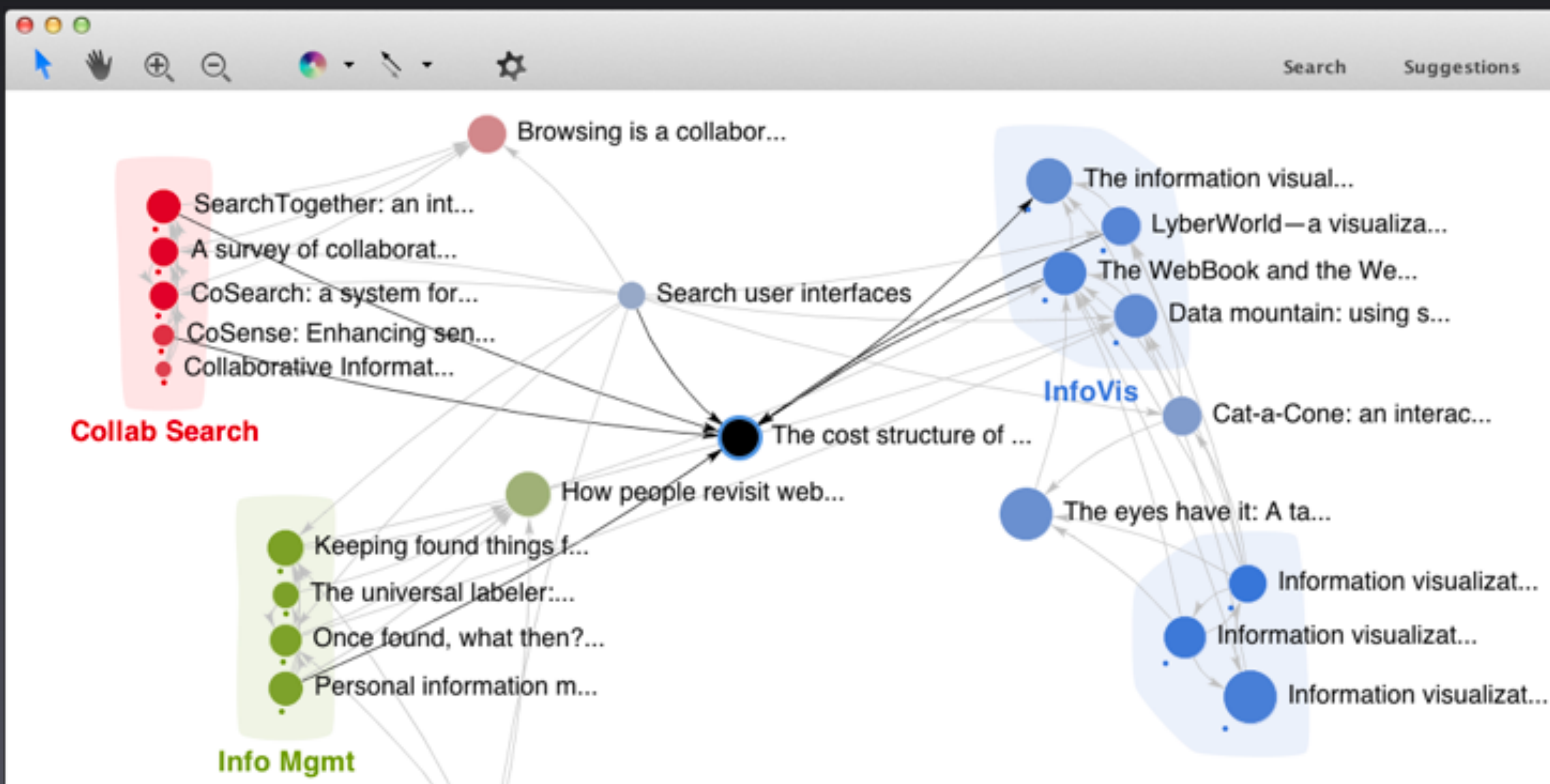
Key Ideas (Recap)

Specify exemplars

Find other relevant nodes (BP)



Apolo's Contributions

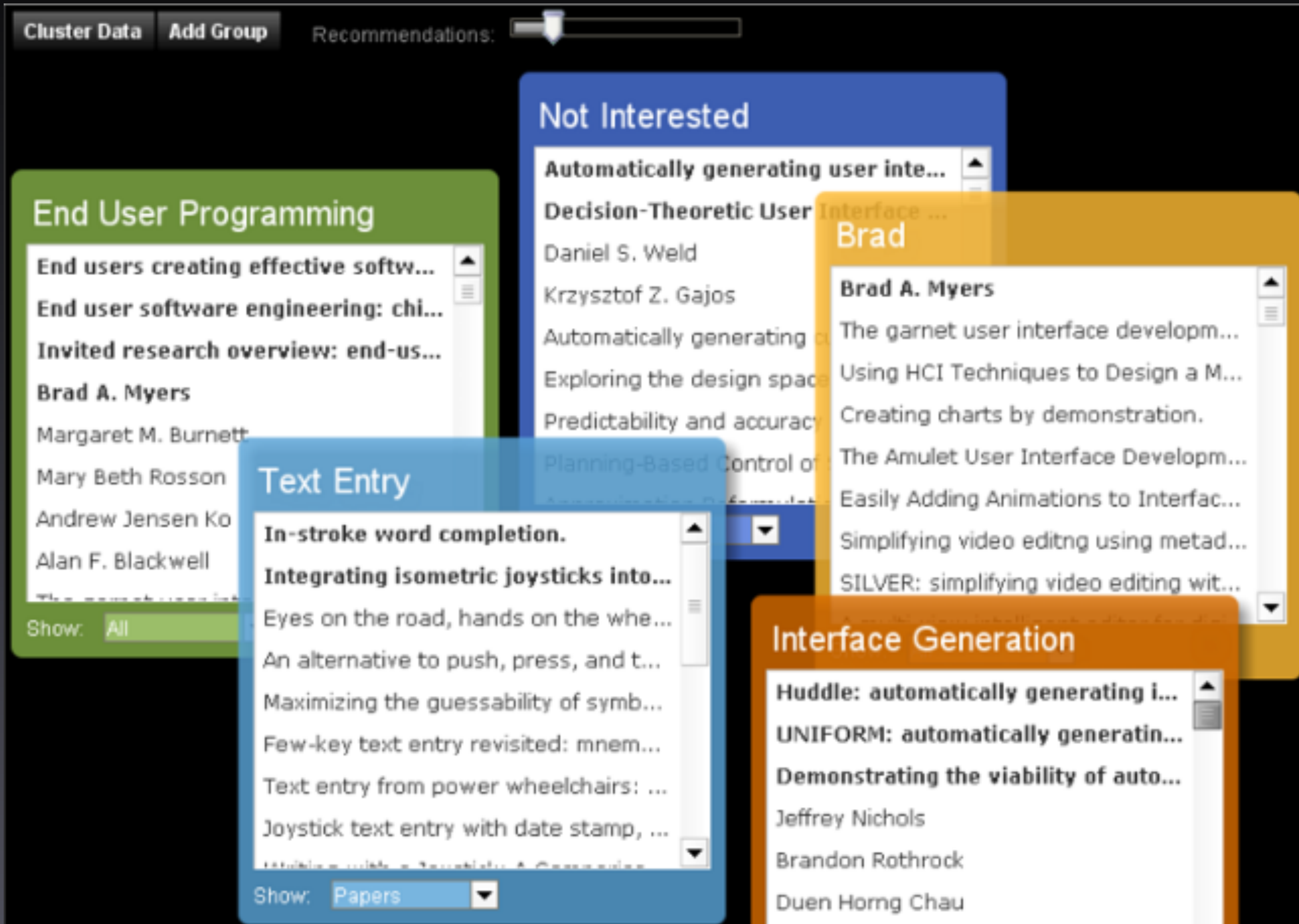
1 **Human** + **Machine**

It was like having a **partnership** with the machine.

Apolo User

2 **Personalized Landscape**

Apolo 2009


Cluster Data
Add Group
Recommendations:
Not Interested
Automatically generating user inte...
Decision-Theoretic User
Daniel S. Weld
Krzysztof Z. Gajos
Automatically generating
Exploring the design spac
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Brad A. Myers
Margaret M. Burnett
Mary Beth Rosson
Andrew Jensen Ko
Alan F. Blackwell
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All
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Brad A. Myers
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Using HCI Techniques to Design a M...
Creating charts by demonstration.
The Amulet User Interface Developm...
Easily Adding Animations to Interfac...
Simplifying video editng using metad...
SILVER: simplifying video editing wit...
Text Entry
In-stroke word completion.
Integrating isometric joysticks into...
Eyes on the road, hands on the whe...
An alternative to push, press, and t...
Maximizing the guessability of symb...
Few-key text entry revisited: mnem...
Text entry from power wheelchairs: ...
Joystick text entry with date stamp, ...
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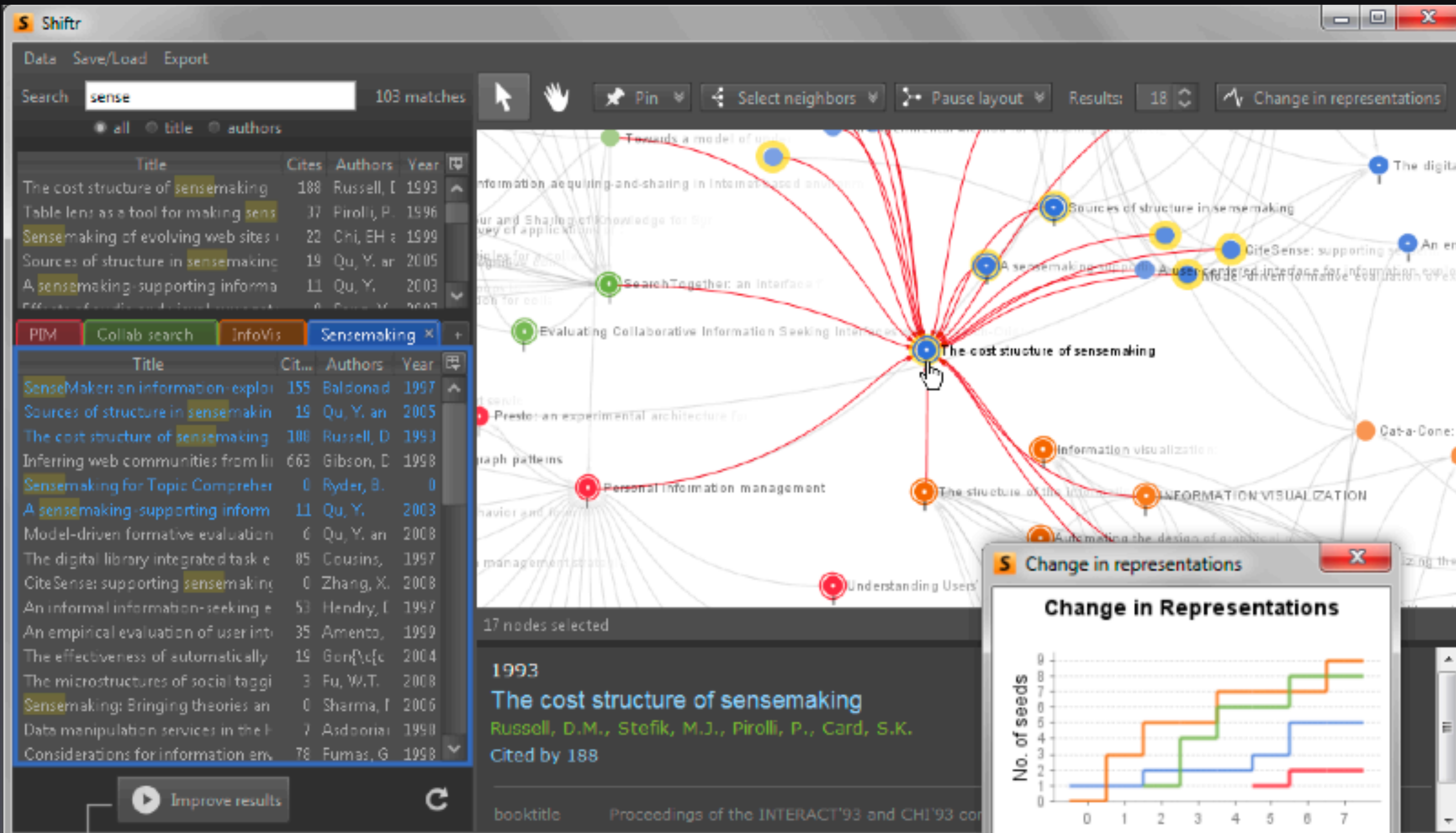
Papers
Interface Generation
Huddle: automatically generating i...
UNIFORM: automatically generatin...
Demonstrating the viability of auto...
Jeffrey Nichols
Brandon Rothrock
Duen Horng Chau

Apolo 2010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Model-driven formative evaluation 0 Qu, Y. an 2008
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CiteSense: supporting sensemaking 0 Zhang, X. 2008
An informal information-seeking e 53 Hendry, D 1997
An empirical evaluation of user int 35 Amento, 1999
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The microstructures of social taggi 3 Fu, W.T. 2008
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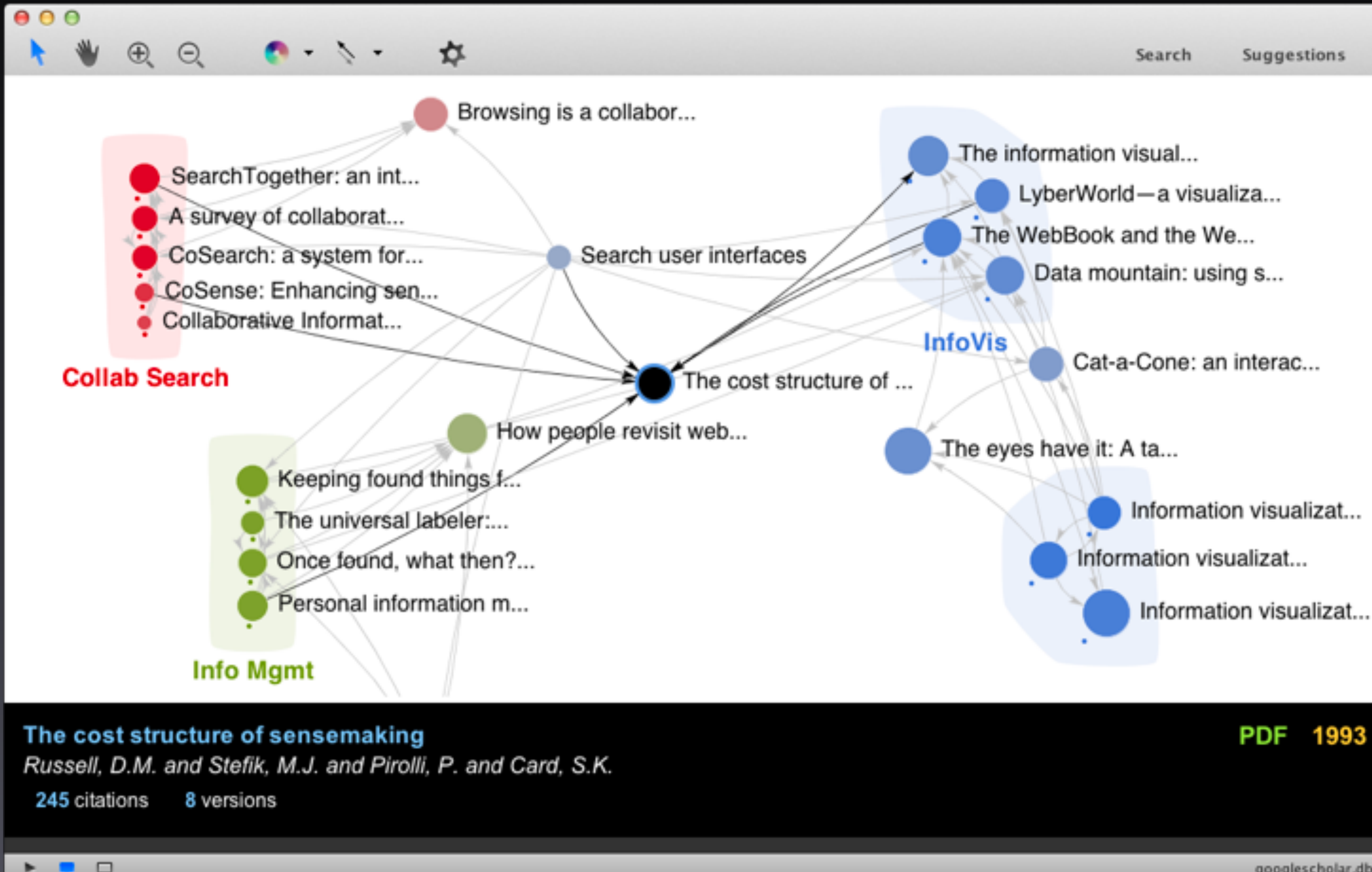
Data manipulation services in the H 7 Asdoorian 1998
Considerations for information env 78 Furnas, G 1998
Improve results
Sources of structure in sensemaking
CiteSense: supporting
SearchTogether: an interface
Evaluating Collaborative Information Seeking Interfaces
The cost structure of sensemaking
Information visualization
Personal information management
INFORMATION VISUALIZATION
Understanding Users
Cat-a-Cone:
17 nodes selected
1993
The cost structure of sensemaking
Russell, D.M., Stefik, M.J., Pirolli, P., Card, S.K.
Cited by 188
booktitle Proceedings of the INTERACT'93 and CHI'93 co
Change in representations
Change in Representations
No. of seeds
Iteration

Apolo 2011

22,000 lines of code. Java 1.6. Swing.
Uses SQLite3 to store graph on disk

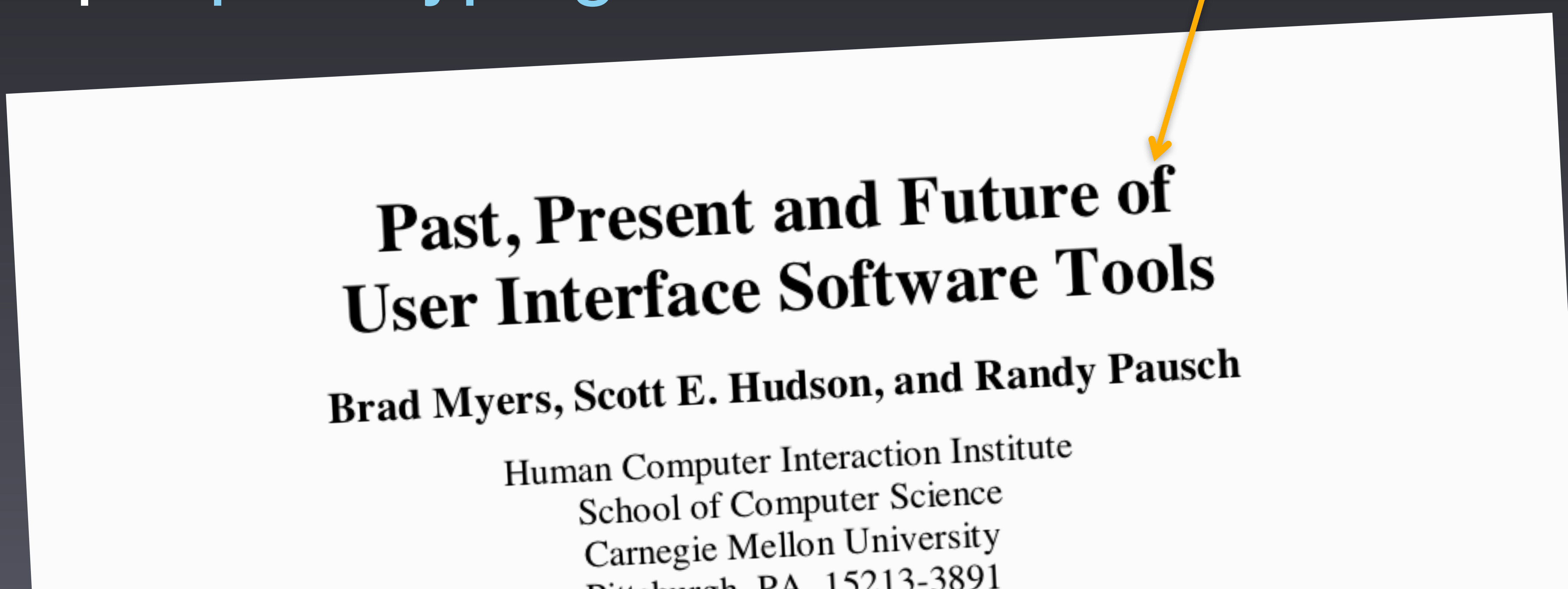


User Study

Used citation network

Task: Find related papers for 2 sections in a survey paper on *user interface*

- Model-based generation of UI
- Rapid prototyping tools



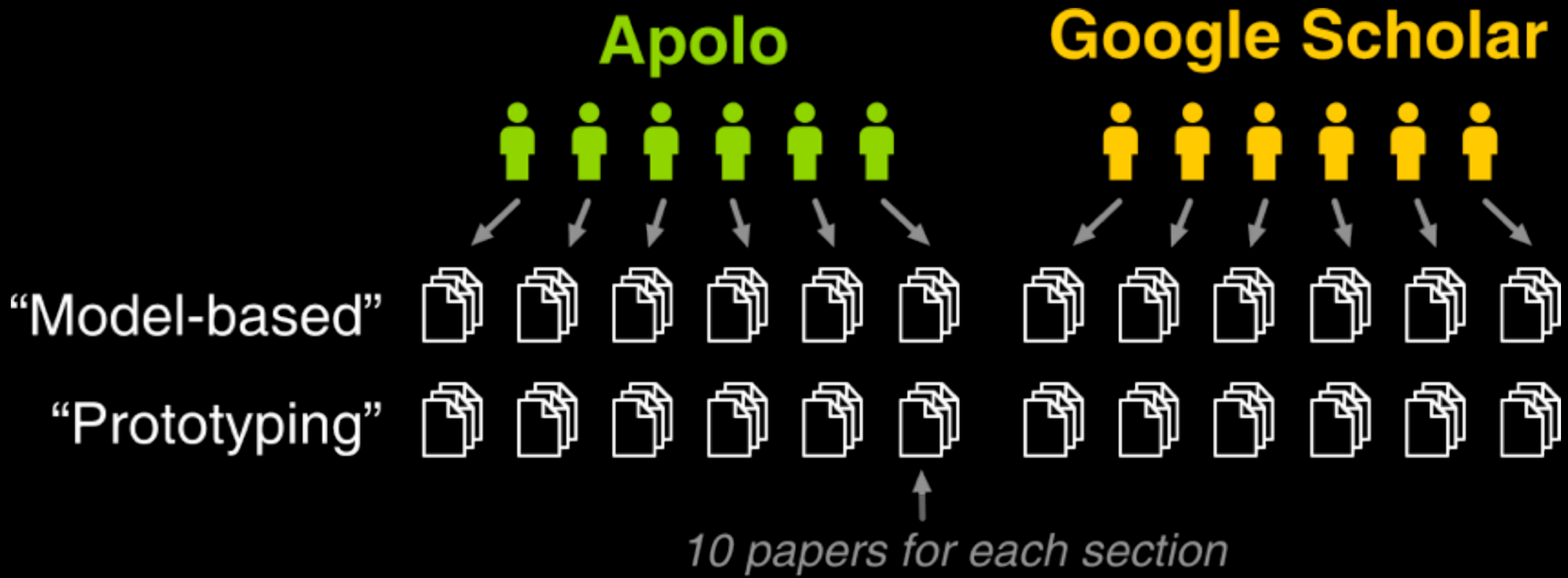
Past, Present and Future of User Interface Software Tools

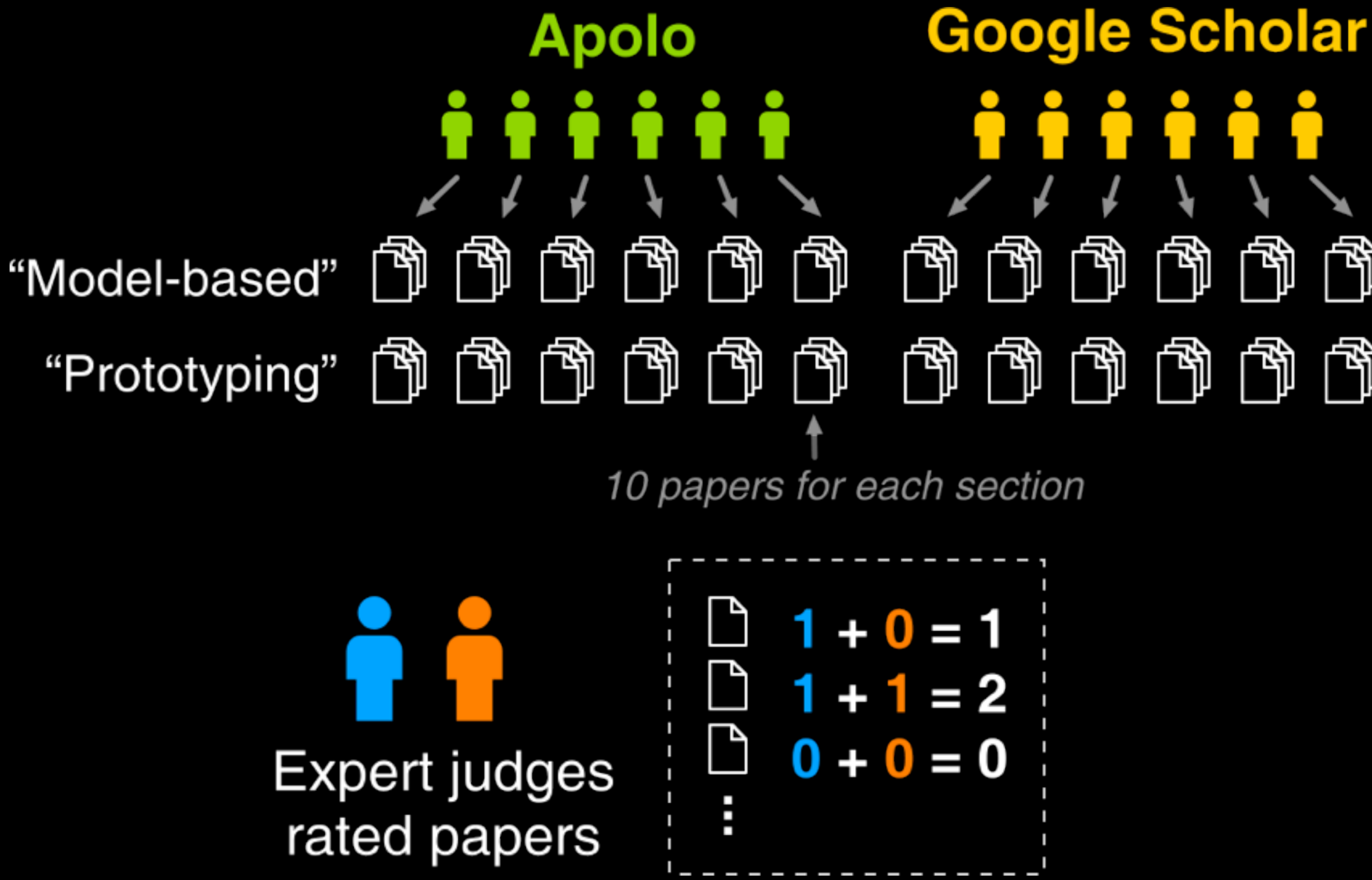
Brad Myers, Scott E. Hudson, and Randy Pausch

Human Computer Interaction Institute
School of Computer Science
Carnegie Mellon University

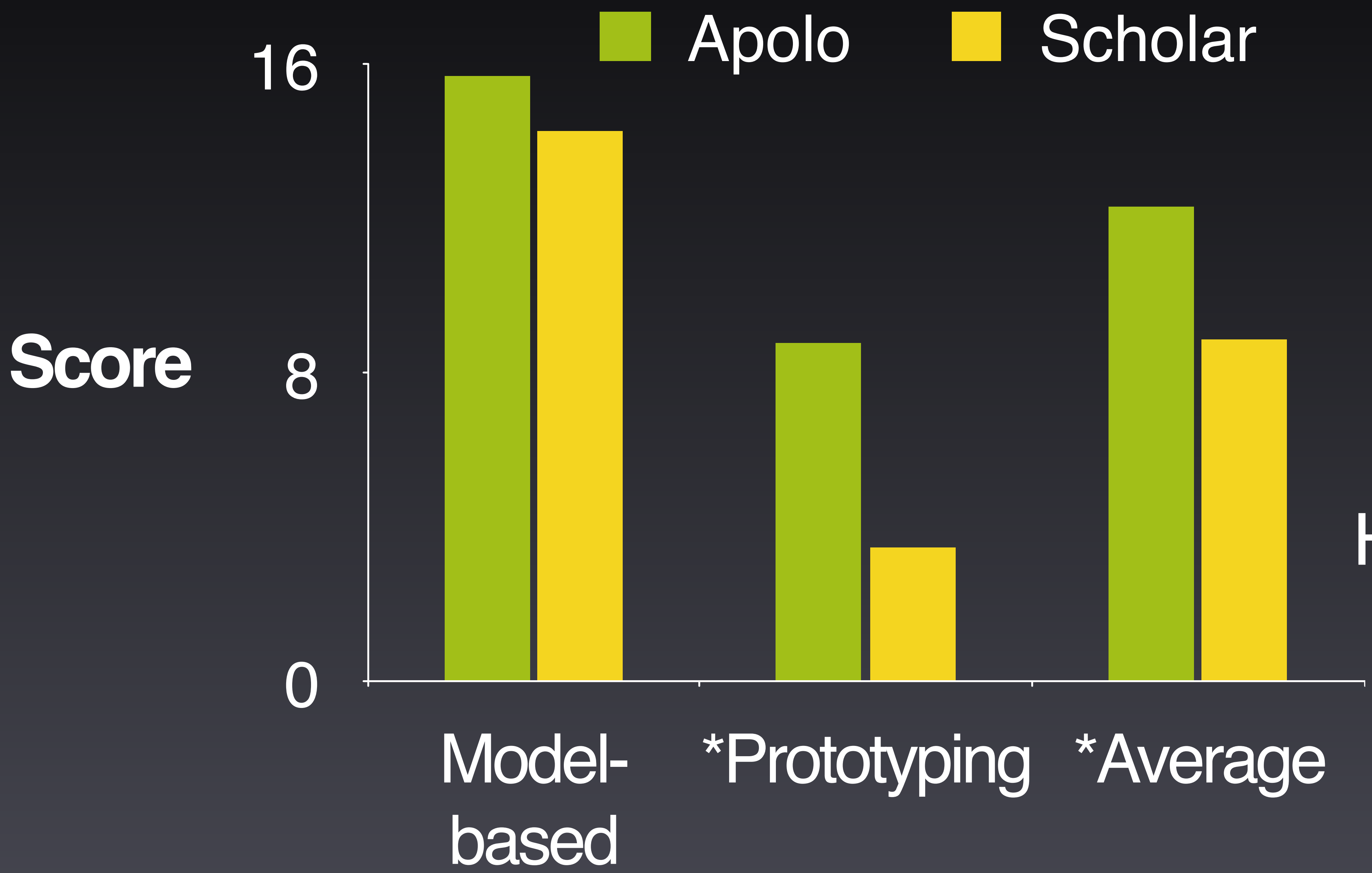
Between subjects design

Participants: **grad student** or **research staff**


Apolo
Google Scholar
"Model-based"
"Prototyping"
10 papers for each section


Apolo
Google Scholar
“Model-based”
“Prototyping”
10 papers for each section
Expert judges rated papers
1 + 0 = 1
1 + 1 = 2
0 + 0 = 0
⋮

Judges' Scores



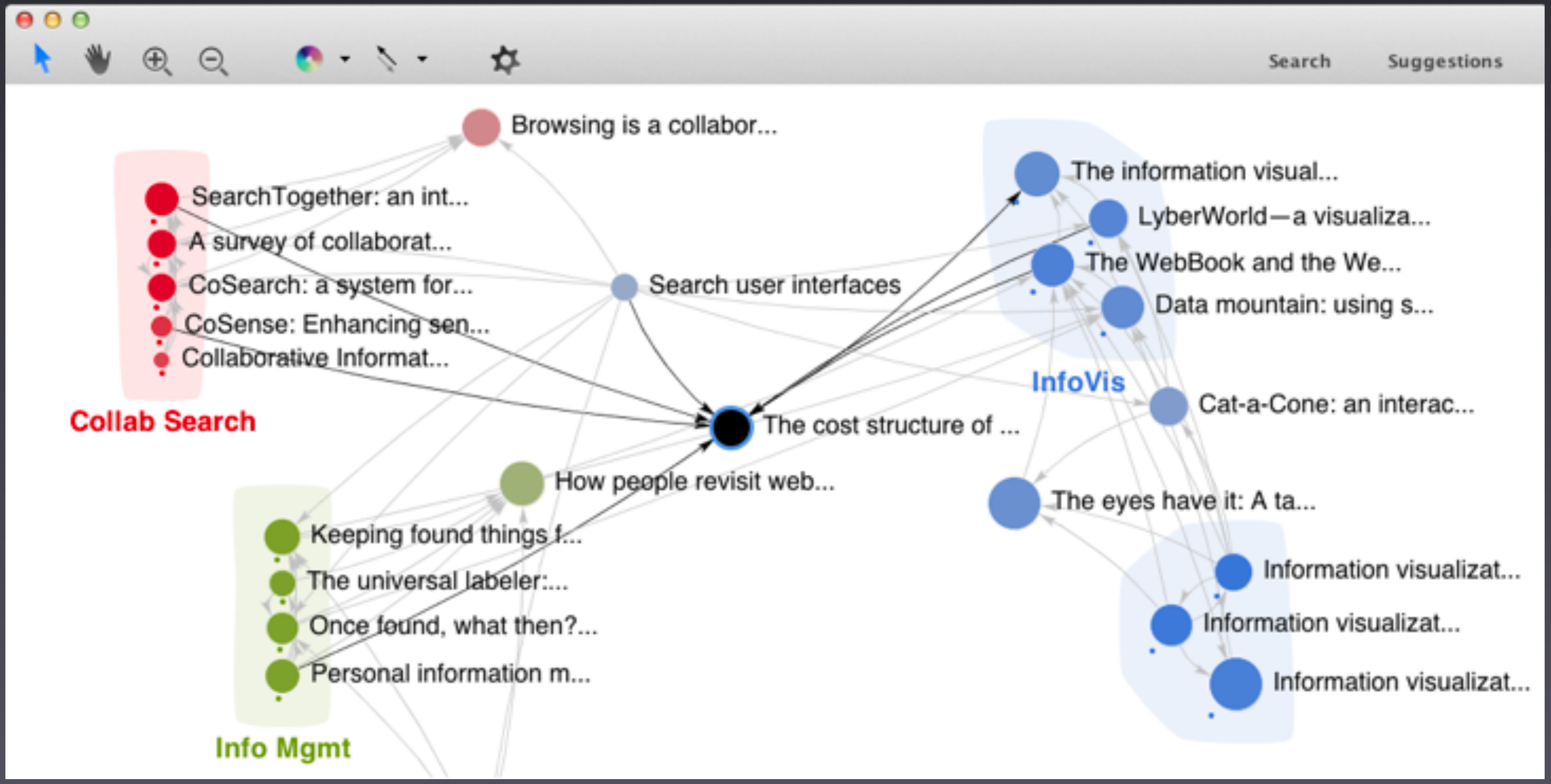
Higher is better.
Apolo wins.

** Statistically significant, by two-tailed t test, p <0.05*

Apolo: Recap

A mixed-initiative approach for exploring and creating personalized landscape for large network data

Apolo = ML + Visualization + Interaction



Practitioners' guide to building (interactive) applications

Think about scalability early

- Identify candidate scalable algorithms early on

Use **iterative** design approach, as in Apolo and industry

- Why? It's hard to get it right the first time
- Create prototype, evaluate, modify prototype, evaluate, ...
- Quick evaluation helps you identify **important fixes early — save you a lot of time overall**

Practitioners’ guide to building (interactive) applications

What kinds of **prototypes**?

- Paper prototype, lo-fi prototype, high-fi prototype

Important to involve **REAL users** as early as possible

- Recruit your friends to try your tools
- Lab study (controlled, as in Apolo)
- Longitudinal study (usage over months)
- Deploy it and see the world’s reaction!

- To learn more:
    - CS 6750 Human-Computer Interaction
    - CS 6455 User Interface Design and Evaluation

If you want to know more about people…

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