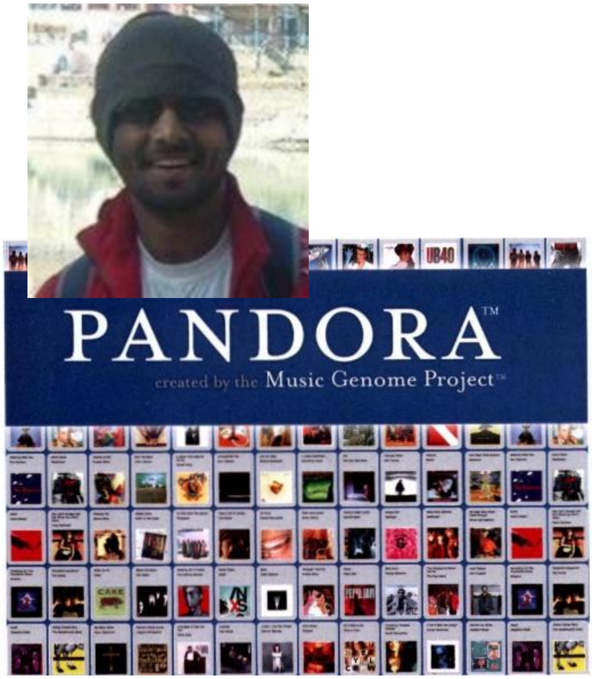
Feb 27, 2014

CSE 6242 / CX 4242

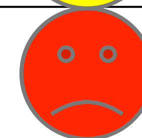
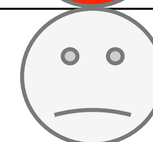
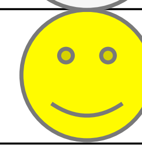
Classification

How to predict a discrete variable?

Based on Parishit Ram's slides



How will I rate "Chopin's 5th Symphony"?

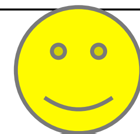
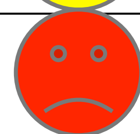
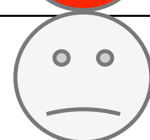
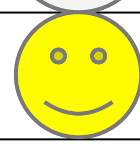
Songs	**Label**
Some nights	
Skyfall	
Comfortably numb	
We are young	
...	...
...	...
Chopin's 5th	???

Classification

What tools do you need for classification?

1. **Data** $S = \{(x_i, y_i)\}_{i = 1,...,n}$
   - x_i represents each example with d attributes
   - y_i represents the label of each example
2. Classification **model** $f_{(a,b,c,....)}$ with some parameters $a, b, c, ...$
   - a model/function maps examples to labels
3. **Loss function** $L(y, f(x))$
   - how to penalize mistakes

Features $x_i = (x_{i1}, \ldots, x_{id})$

Song name	**Label**	**Artist**	**Length**	**...**
Some nights		Fun	4:23	...
Skyfall		Adele	4:00	...
Comf. numb		Pink Fl.	6:13	...
We are young		Fun	3:50	...
...	...	...	...	...
...	...	...	...	...
Chopin's 5th	??	Chopin	5:32	...

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Training a classifier (building the "model")

Q: How do you learn appropriate values for parameters *a, b, c, …* such that

- (Part I) $y_i = f_{(a,b,c,\ldots.)}(x_i), i = 1, \ldots, n$
  - Low/no error on the training set
- (Part II) $y = f_{(a,b,c,\ldots.)}(x)$, for any new x
  - Low/no error on future queries (songs)

Possible A: Minimize $\sum_{i=1}^{n} L(y_i, f_{(a,b,c,..)}(x_i))$ with respect to *a, b, c,…*

Classification loss function

Most common loss: **0-1 loss function**

$$L_{0-1}(y, f(x)) = \mathbb{I}(y \neq f(x))$$

More general loss functions are defined by a *m x m* cost matrix *C* such that

$$L(y, f(x)) = C_{ab}$$

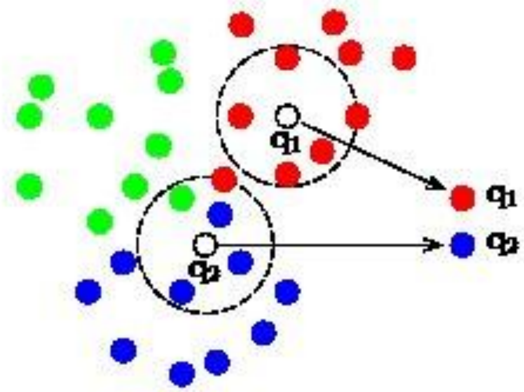
where $y = a$ and $f(x) = b$

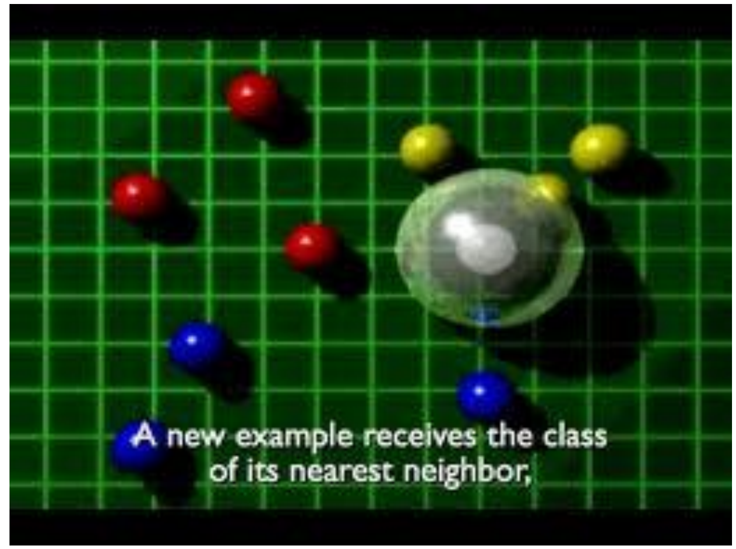
Class	T0	T1
P0	0	C_{10}
P1	C_{01}	0

T0 (true class 0), **T1** (true class 1)

P0 (predicted class 0), **P1** (predicted class 1)

k-Nearest-Neighbor Classifier





The classifier:

f(x) = majority label of the *k* nearest neighbors (NN) of *x*

Model parameters:

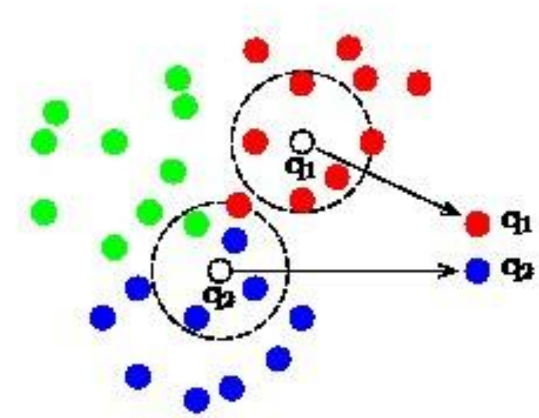
- number of neighbors *k*
- distance function *d(.,.)*

k-Nearest-Neighbor Classifier

If *k* and *d(.,.)* are fixed

Things to learn: ?

How to learn them: ?



If *d(.,.)* is fixed, but you can change *k*

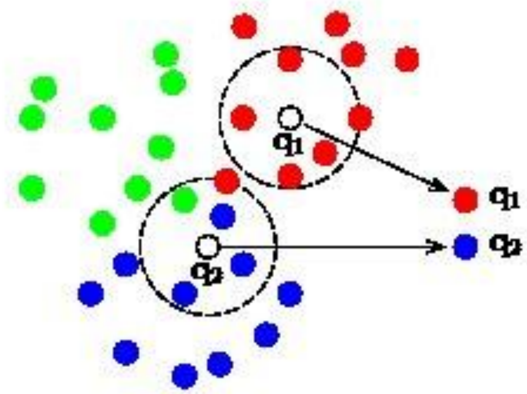
Things to learn: ?

How to learn them: ?

$x_i = (x_{i1}, \dots, x_{id}); y_i = \{1, \dots, m\}$

k-Nearest-Neighbor Classifier

If *k* and *d(.,.)* are fixed



Things to learn: Nothing

How to learn them: N/A

If *d(.,.)* is fixed, but you can change *k*

Things to learn: Nothing

How to learn them: N/A

Selecting *k*: Try different values of *k* on some hold-out set

Cross-validation

Find the best performing *k*

1. Hold out a part of the training data
2. Train on the rest of the data
3. Evaluate the training error on the holdout set
4. Do this multiple times, once for each k, and pick the *k* with best performance
    - with respect to the error (on hold-out set) averaged over all hold-out sets

Cross-validation: Holdout sets

Leave-one-out cross-validation (LOO-CV)

- hold out sets of size 1

K-fold cross-validation

- hold sets of size (n / K)
- K = 10 is most common (i.e., 10 fold CV)

$x_i = (x_{i1}, \dots, x_{id}); y_i = \{1, \dots, m\}$

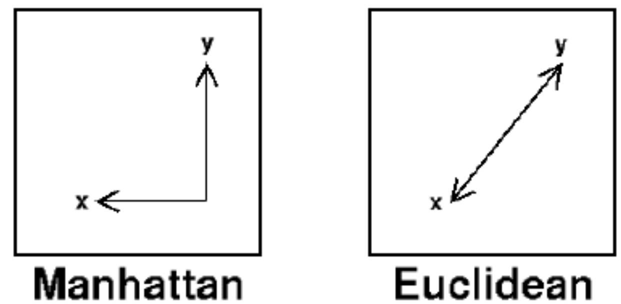
k-Nearest-Neighbor Classifier

If k is fixed, but you can change *d(.,.)*

Things to learn: ?

How to learn them: ?

Cross-validation: ?



Possible distance functions:

- Euclidean distance: $\|x_i - x_j\|_2 = \sqrt{(x_i - x_j)^\top (x_i - x_j)}$
- Manhattan distance: $\|x_i - x_j\|_1 = \sum_{l=1}^{d} |x_{il} - x_{jl}|$
- …

$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$

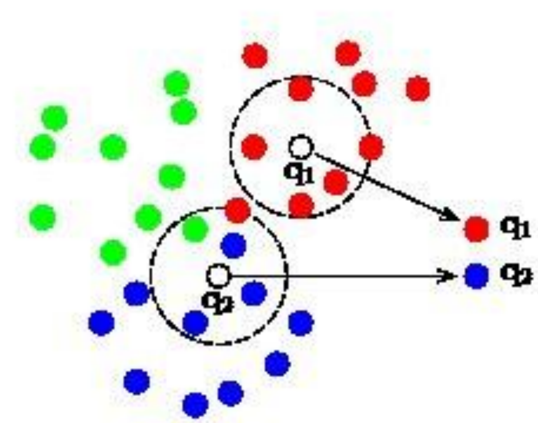
k-Nearest-Neighbor Classifier

If *k* is fixed, but you can change *d(.,.)*

Things to learn: distance function *d(.,.)*

How to learn them: optimization

Cross-validation: any regularizer you have on your distance function



Summary on k-NN classifier

- Advantages
  - No learning (unless you are learning the distance functions)
  - quite powerful in practice (and has theoretical guarantees as well)
- Caveats
  - Computationally expensive at test time

Reading material:

- ESL book, Chapter 13.3
  http://www-stat.stanford.edu/~tibs/ElemStatLearn/printings/ESLII_print10.pdf
- Le Song's slides on kNN classifier
  http://www.cc.gatech.edu/~lsong/teaching/CSE6740/lecture2.pdf

Points about cross-validation

Requires extra computation, but gives you information about expected test error

LOO-CV:

- Advantages
  - Unbiased estimate of test error (especially for small n)
  - Low variance
- Caveats
  - Extremely time consuming

Points about cross-validation

K-fold CV:

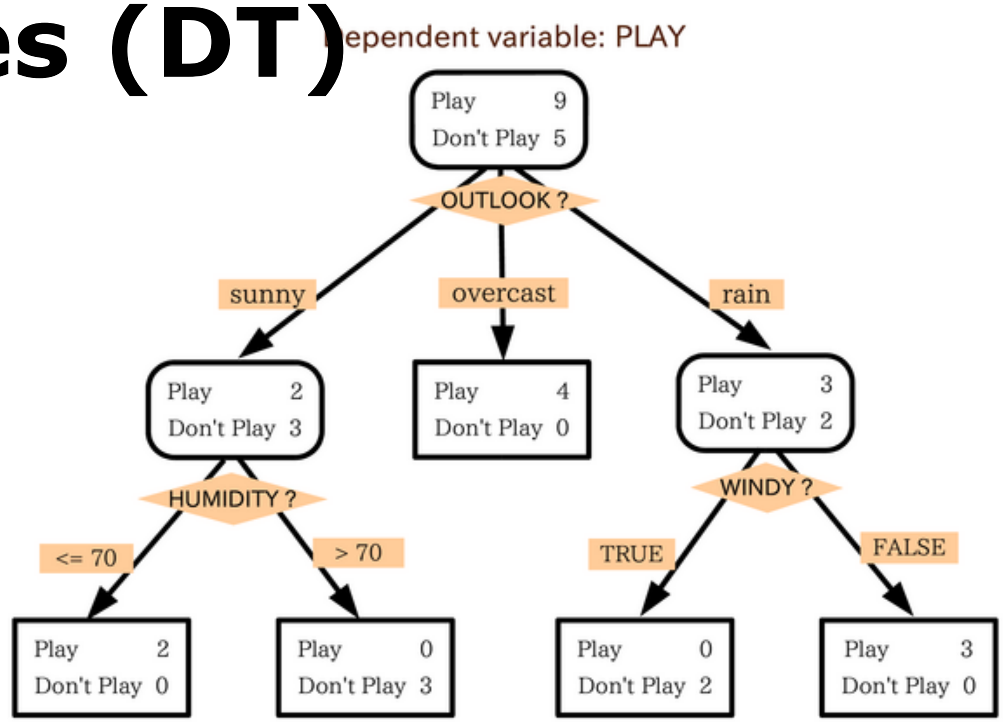
- Advantages
  - More efficient than LOO-CV
- Caveats
  - *K* needs to be large for low variance
  - Too small *K* leads to **under-use** of data, leading to higher bias
- Usually accepted value ***K* = 10**

Reading material:

- ESL book, Chapter 7.10
  http://www-stat.stanford.edu/~tibs/ElemStatLearn/printings/ESLII_print10.pdf
- Le Song's slides on CV
  http://www.cc.gatech.edu/~lsong/teaching/CSE6740/lecture13-cv.pdf

$$x_i = (x_{i1}, \dots, x_{id}); y_i = \{1, \dots, m\}$$

Decision trees (DT)



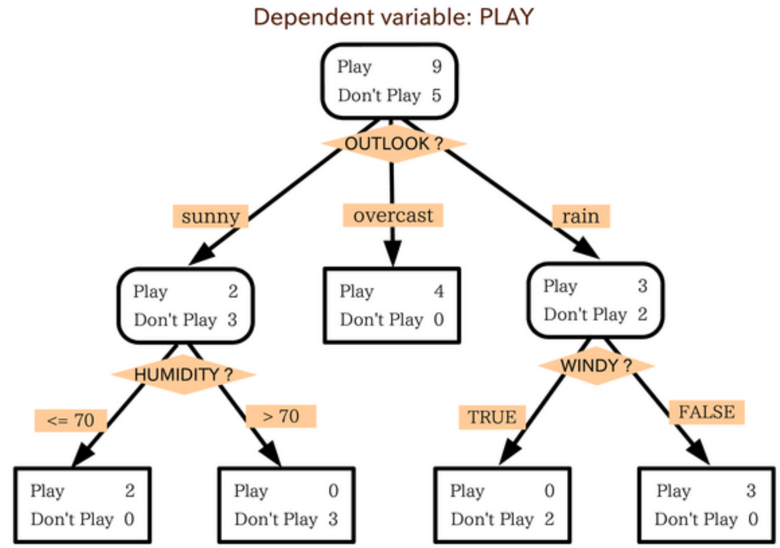
The classifier:

$f_T(x)$ is the majority class in the leaf in the tree T containing x

Model parameters: The tree structure and size

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Decision trees



Things to learn: ?

How to learn them: ?

Cross-validation: ?

$x_i = (x_{i1}, \dots, x_{id}); y_i = \{1, \dots, m\}$

Decision trees

Things to learn: the tree structure

How to learn them: (greedily) minimize the overall classification loss

Cross-validation: finding the best sized tree with *K*-fold cross-validation

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Learning the tree structure

Pieces:

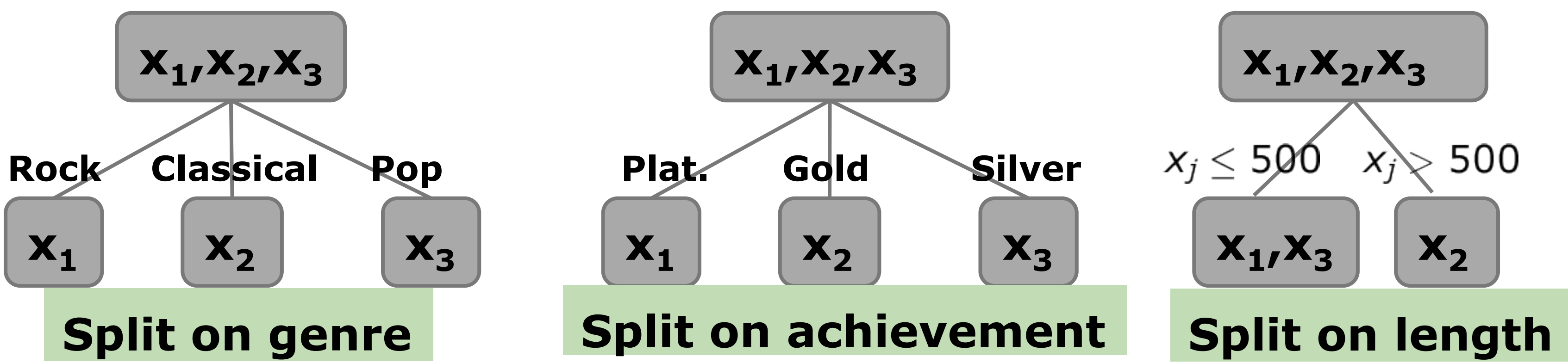
1. best split on the chosen attribute
2. best attribute to split on
3. when to stop splitting
4. cross-validation

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Choosing the split

Split types for a selected attribute *j*:

1. Categorical attribute (e.g. `genre')
   x_{1j} = *Rock*, x_{2j} = *Classical*, x_{3j} = *Pop*
2. Ordinal attribute (e.g. `achievement')
   x_{1j}=*Gold*, x_{2j}=*Platinum*, x_{3j}=*Silver*
3. Continuous attribute (e.g. song length)
   $x_{1j} = 235$, $x_{2j} = 543$, $x_{3j} = 378$



$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Choosing the split

At a node T for a given attribute d,

select a split s as following:

$$min_s \; loss(T_L) + loss(T_R)$$

where $loss(T)$ is the loss at node T

Node loss functions:

- Total loss: $\sum_{x_i \in T} L(y_i, f_T(x_i)) - \sum_{c \in T} p_{cT} \log p_{cT}$
- Cross-entropy: where p_{cT} is the proportion of class c in node T

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Choosing the attribute

Choice of attribute:

1. Attribute providing the maximum improvement in training loss
2. Attribute with maximum information gain

$$\min_{j \in \{1,\ldots,d\}} \min_{s(j)} loss(T_L) + loss(T_R)$$

$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$

When to stop splitting?

1. Homogenous node (all points in the node belong to the same class OR all points in the node have the same attributes)
2. Node size less than some threshold
3. Further splits provide no improvement in training loss ($loss(T) <= loss(T_L) + loss(T_R)$)

$$x_i = (x_{i1}, \ldots, x_{id}); y_i = \{1, \ldots, m\}$$

Controlling tree size

In most cases, you can drive training error to zero (how? is that good?)

What is wrong with really deep trees?

- Really high "variance"

What can be done to control this?

- *Regularize* the tree complexity
  - Penalize complex models and prefers simpler models (why?)

Look at Le Song's slides on the decomposition of error in bias and variance of the estimator http://www.cc.gatech.edu/~lsong/teaching/CSE6740/lecture13-cv.pdf

Summary on decision trees

- Advantages
  - Easy to implement
  - Interpretable
  - Very fast test time
  - Can work seamlessly with mixed attributes
  - ** Works quite well in practice
- Caveats
  - Can be too simplistic
  - Training can be very expensive
  - Cross-validation is hard (and plain annoying)

Final words on decision trees

Reading material:

- ESL book, Chapter 9.2
  http://www-stat.stanford.edu/~tibs/ElemStatLearn/printings/ESLII_print10.pdf
- Le Song's slides
  http://www.cc.gatech.edu/~lsong/teaching/CSE6740/lecture6.pdf