http://poloclub.gatech.edu/cse6242

CSE6242 / CX4242: Data & Visual Analytics

Text Analytics (Text Mining)

Concepts, Algorithms, LSI/SVD

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Partly based on materials by
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Text is everywhere

We use documents as primary information artifact in our lives

Our access to documents has grown tremendously thanks to the Internet

- **WWW**: webpages, Twitter, Facebook, Wikipedia, Blogs, ...
- **Digital libraries**: Google books, ACM, IEEE, ...
- Lyrics, closed caption... (youtube)
- Police case reports
- legislation (law)
- reviews (products, rotten tomatoes)
- medical reports (EHR - electronic health records)
- job descriptions

Big (Research) Questions

... in understanding and gathering information from text and document collections

- establish authorship, authenticity; plagiarism detection
- classification of genres for narratives (e.g., books, articles)
- tone classification; sentiment analysis (online reviews, twitter, social media)
- code: syntax analysis (e.g., find common bugs from students' answers)

Popular NLP libraries

- **Stanford NLP**
- **OpenNLP**
- **NLTK** (python)

tokenization, sentence segmentation, part-of-speech tagging, named entity extraction, chunking, parsing

Outline

- **Preprocessing** (e.g., stemming, remove stop words)
- **Document representation** (most common: bag-of-words model)
- **Word importance** (e.g., word count, TF-IDF)
- **Latent Semantic Indexing** (find "concepts" among documents and words), which helps with **retrieval**

To learn more: Prof. Jacob Eisenstein's
CS 4650/7650 Natural Language Processing

Stemming

Reduce words to their **stems** (or base forms)

> **Words**: compute, computing, computer, ...
>
> **Stem**: comput

Several classes of algorithms to do this:

- Stripping suffixes, lookup-based, etc.

http://en.wikipedia.org/wiki/Stemming
Stop words: http://en.wikipedia.org/wiki/Stop_words

Bags-of-words model

Represent each **document** as a **bag of words**, ignoring words' ordering. Why? For simplicity.

- Unstructured text -> a vector of numbers
- e.g., docs: "I like visualization", "I like data".
  - "I": 1,
  - "like": 2,
  - "data": 3,
  - "visualization": 4
- "I like visualization" -> [1, 1, 0, 1]
- "I like data" -> [1, 1, 1, 0]

TF-IDF

(**a word's importance score** in a **document**, among **N documents**)

When to use it? Everywhere you use "word count", you may use TF-IDF.

TF: term frequency
= #appearance a document
(high, if terms appear many times in this document)

IDF: inverse document frequency
= log(**N** / #document containing that term)
(penalize "common" words appearing in almost any documents)

Final score = TF * IDF
(higher score -> more important)

Example: http://en.wikipedia.org/wiki/Tf–idf#Example_of_tf.E2.80.93idf

Vector Space Model

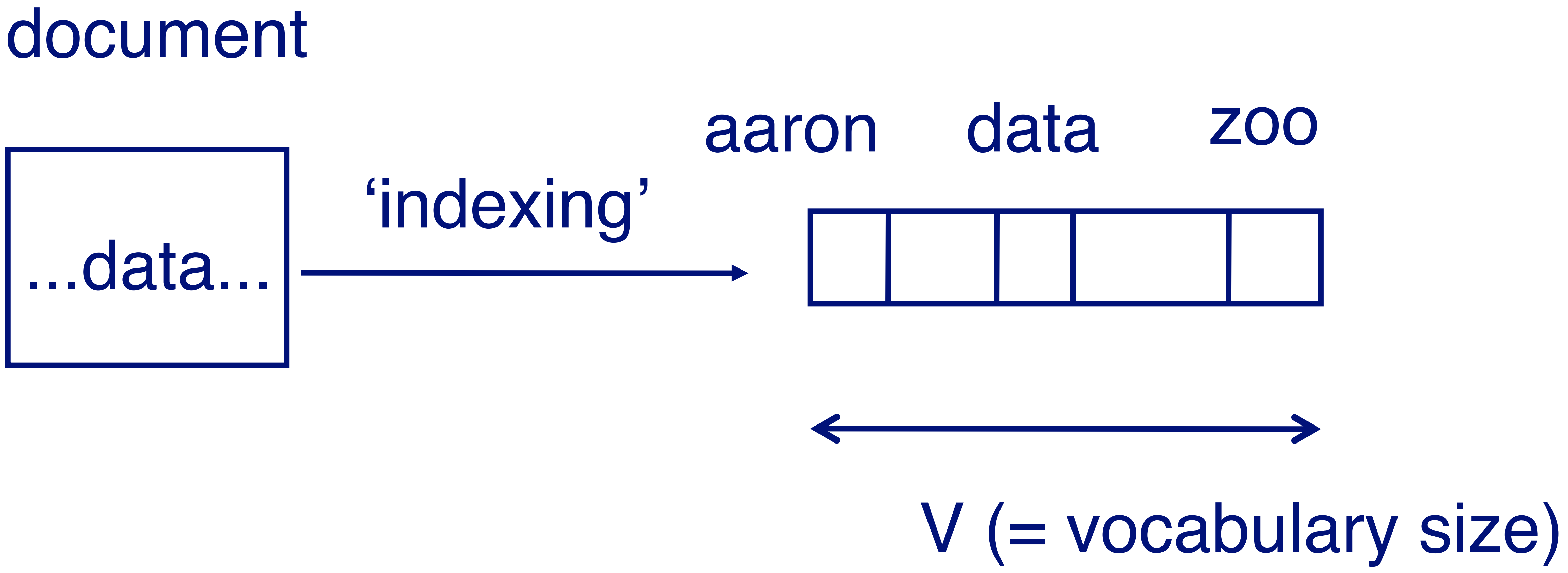
Each document -> vector

Each query -> vector

Search for documents -> find “similar” vectors

Vector Space Model and Clustering

- Main idea:

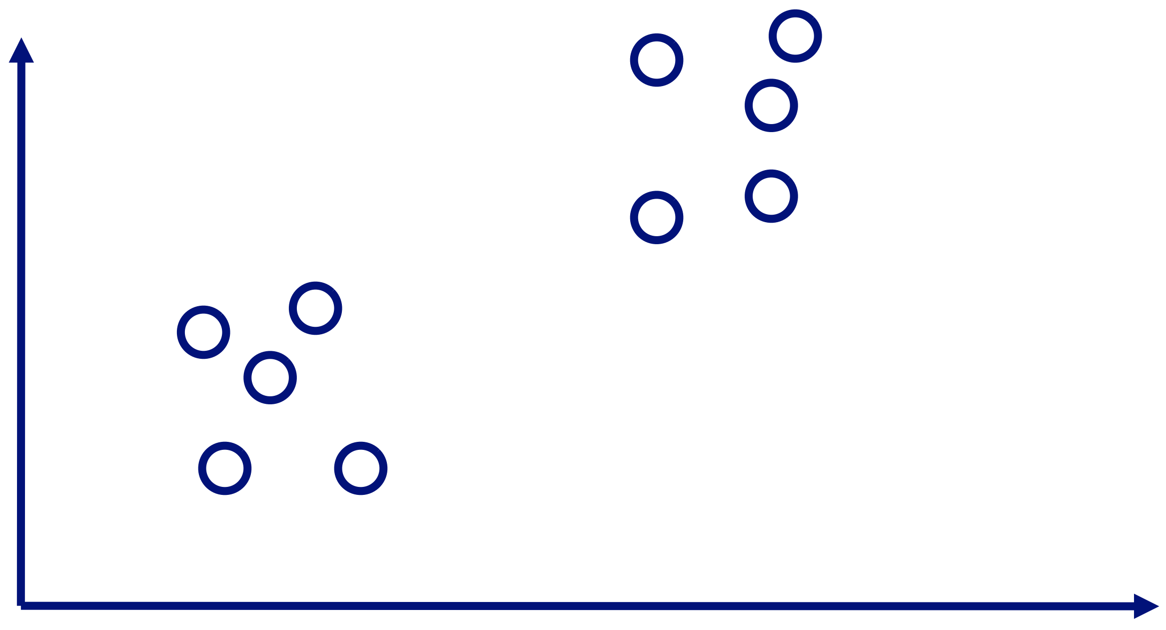


Outline - detailed

- main idea
- cluster search
- cluster generation
- evaluation

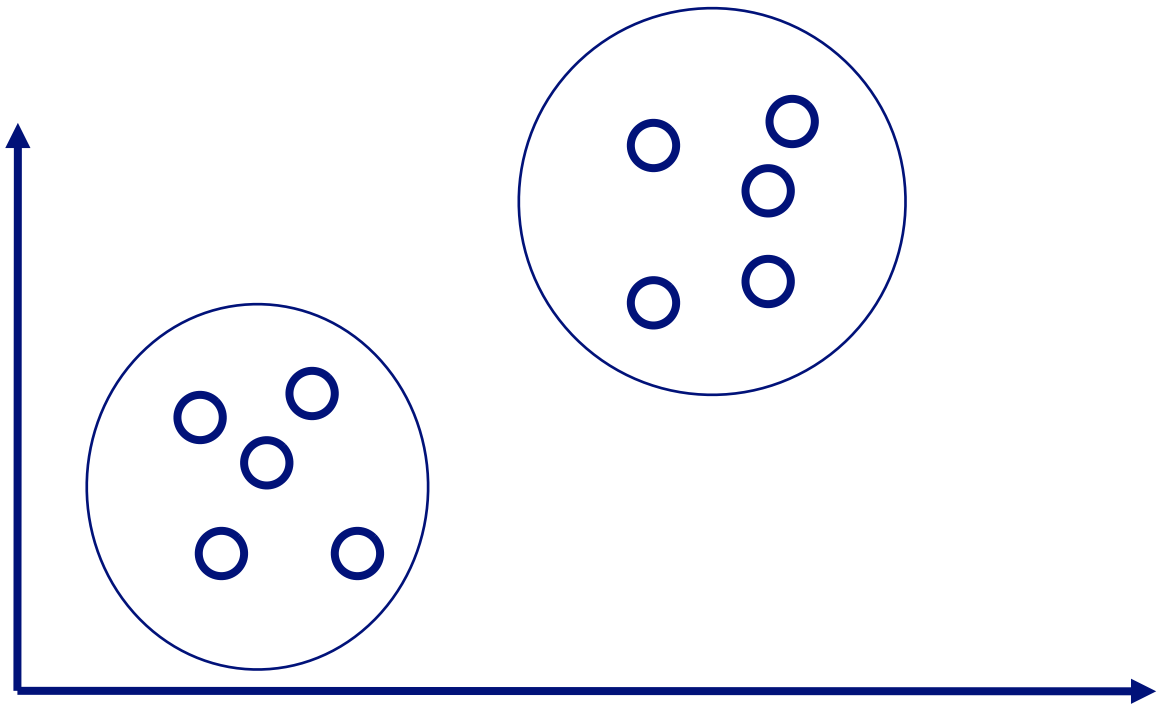
Cluster generation

- Problem:
  - given N points in V dimensions,
  - group them



Cluster generation

- Problem:
  - given N points in V dimensions,
  - group them



Cluster generation

We need

- Q1: document-to-document similarity
- Q2: document-to-cluster similarity

Cluster generation

Q1: document-to-document similarity

(recall: ‘bag of words’ representation)

- D1: {‘data’, ‘retrieval’, ‘system’}
- D2: {‘lung’, ‘pulmonary’, ‘system’}
- distance/similarity functions?

Cluster generation

A1: # of words in common

A2: .... normalized by the vocabulary sizes

A3: .... etc

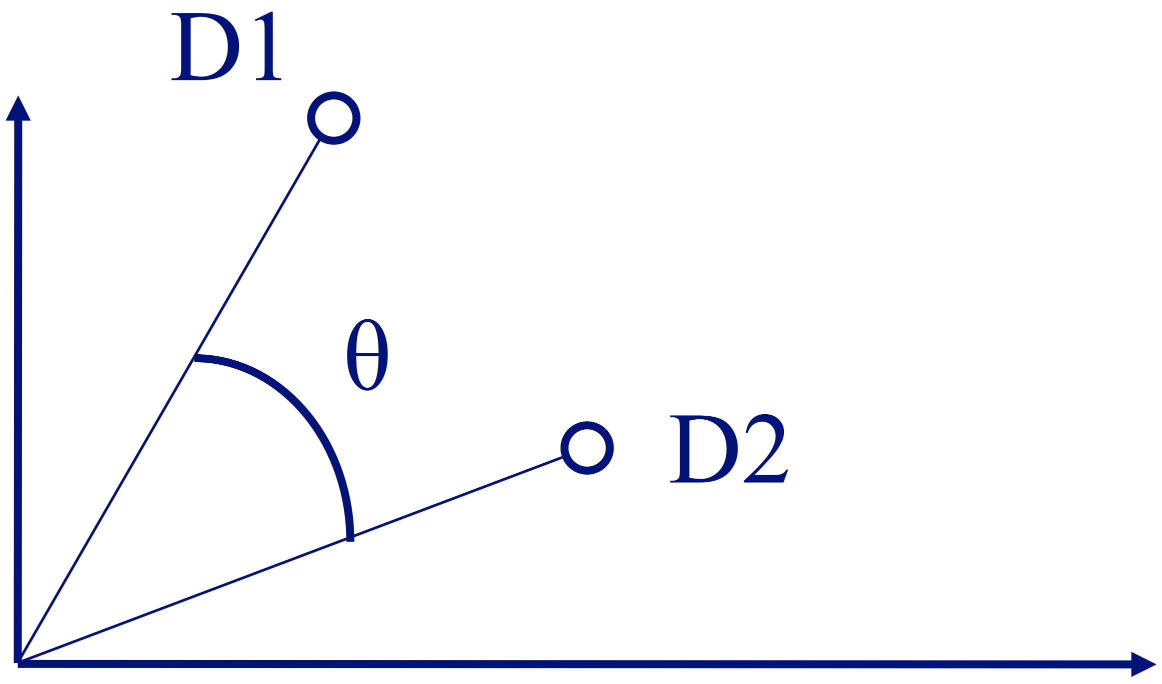
About the same performance - prevailing one:
cosine similarity

Cluster generation

cosine similarity:

$$\text{similarity}(D1, D2) = \cos(\theta) =$$

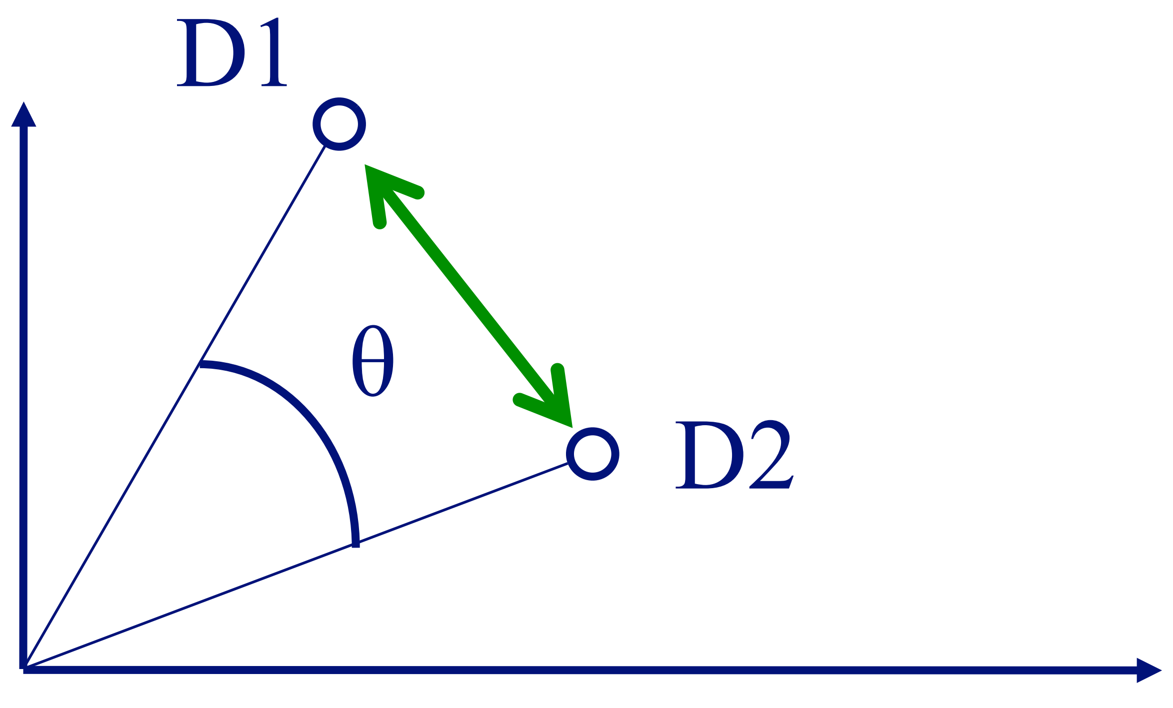
$$\text{sum}(v_{1,i} * v_{2,i}) / [\text{len}(v_1) * \text{len}(v_2)]$$



Cluster generation

cosine similarity - observations:

- related to the Euclidean distance
- weights $v_{i,j}$: according to tf/idf



Cluster generation

tf (‘term frequency’)

- high, if the term appears very often in this document.

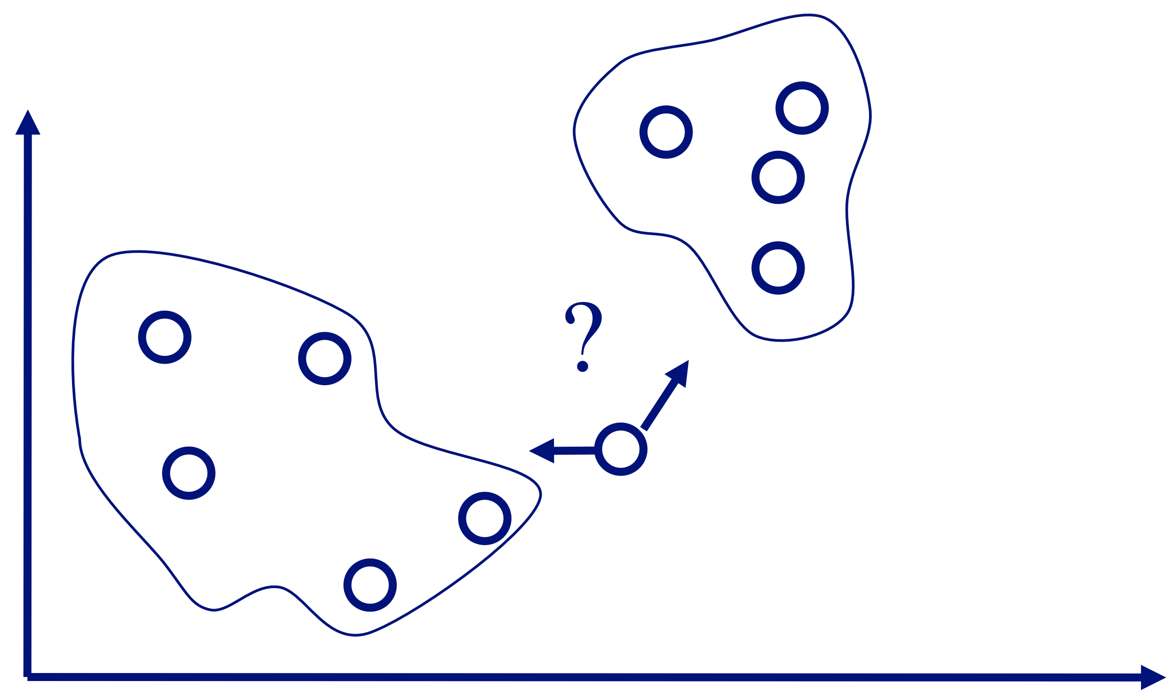
idf (‘inverse document frequency’)

- penalizes ‘common’ words, that appear in almost every document

Cluster generation

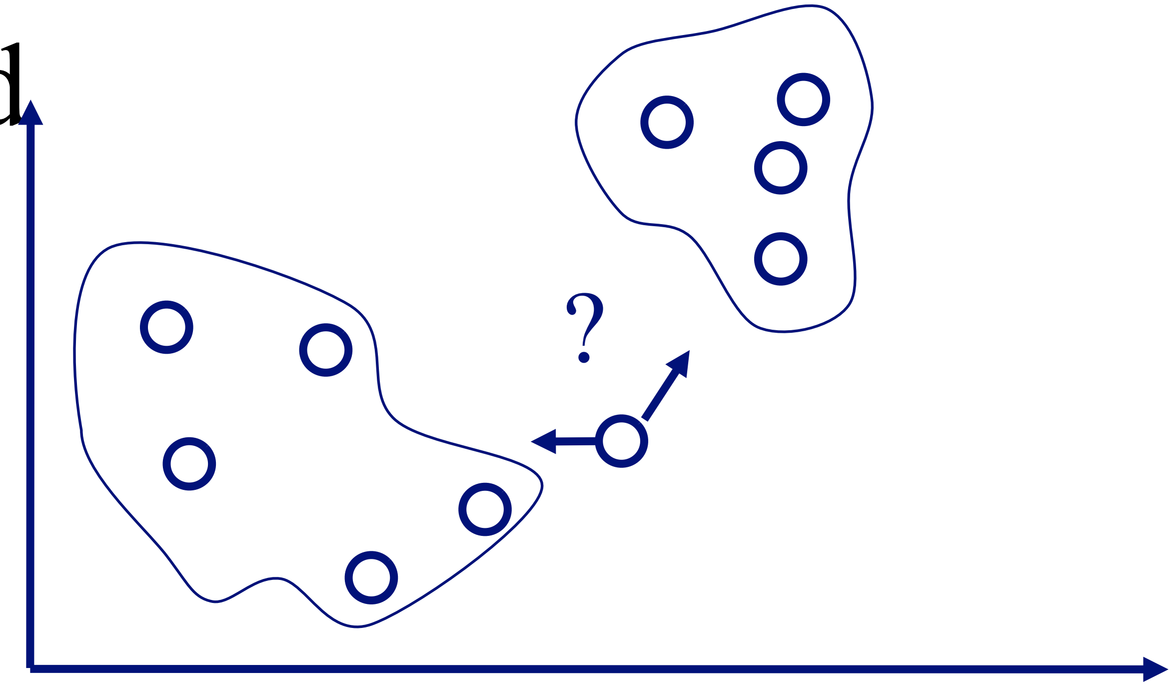
We need

- Q1: document-to-document similarity
- Q2: document-to-cluster similarity



Cluster generation

- A1: min distance (‘single-link’)
- A2: max distance (‘all-link’)
- A3: avg distance (gives same cluster ranking as A4, but different values)
- A4: distance to centroid



Cluster generation

- A1: min distance (‘single-link’)
  - leads to elongated clusters
- A2: max distance (‘all-link’)
  - many, small, tight clusters
- A3: avg distance
  - in between the above
- A4: distance to centroid
  - fast to compute

Cluster generation

We have

- document-to-document similarity
- document-to-cluster similarity

Q: How to group documents into ‘natural’ clusters

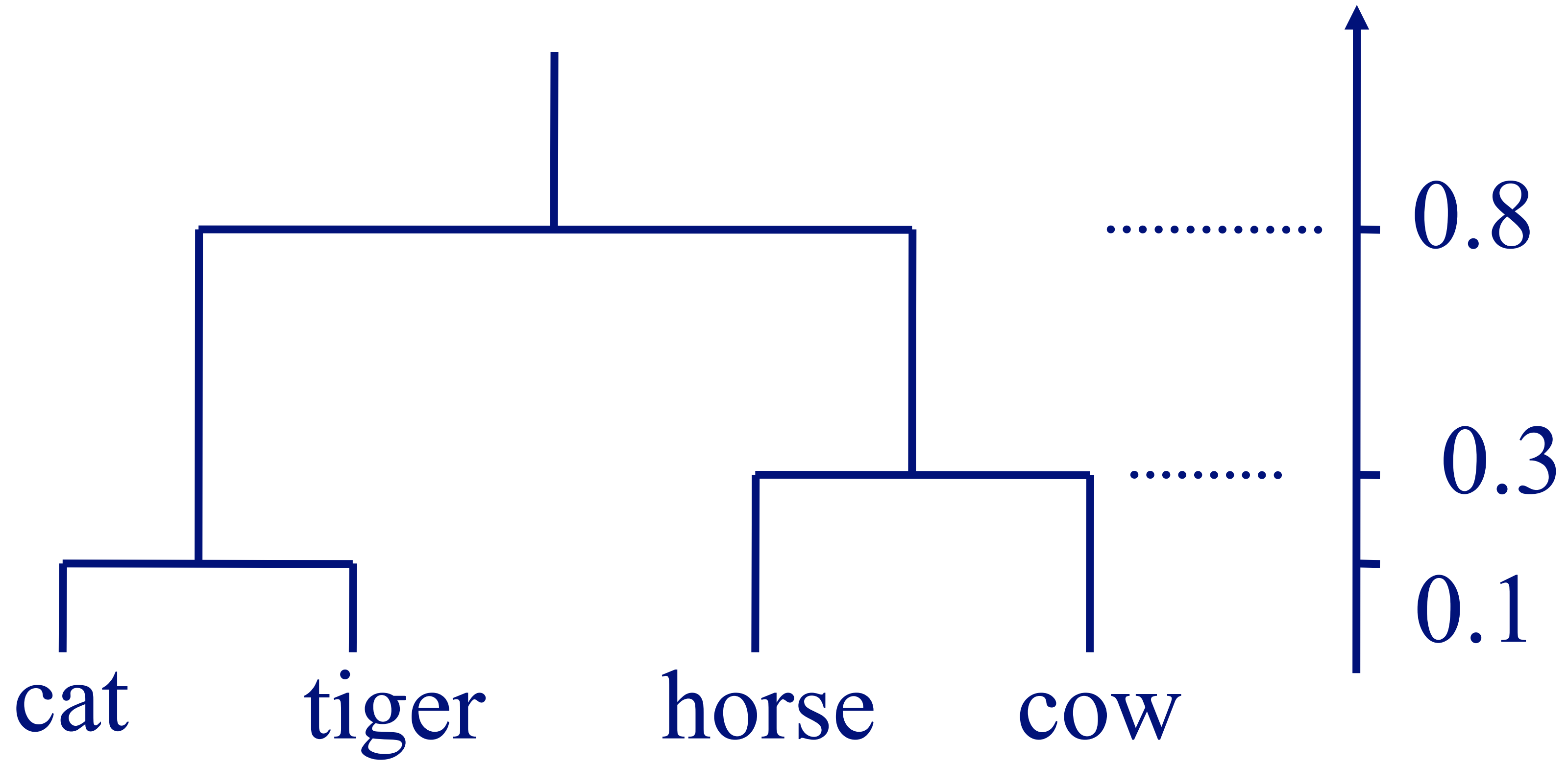
Cluster generation

A: *many-many* algorithms - in two groups [VanRijsbergen]:

- theoretically sound (O(N^2))
  - independent of the insertion order
- iterative (O(N), O(N log(N))

Cluster generation - ‘sound’ methods

- Approach#1: dendrograms - create a hierarchy (bottom up or top-down) - choose a cut-off (how?) and cut



Cluster generation - ‘sound’ methods

- Approach#2: min. some statistical criterion (eg., sum of squares from cluster centers)
  - like ‘k-means’
  - but how to decide ‘k’?

Cluster generation

one way to estimate # of clusters k: the ‘cover coefficient’ [Can+] ~ SVD

LSI - Detailed outline

- LSI
  - problem definition
  - main idea
  - experiments

Information Filtering + LSI

- [Foltz+,'92] Goal:
  - users specify interests (= keywords)
  - system alerts them, on suitable news-documents
- Major contribution:
  LSI = Latent Semantic Indexing
  - latent ('hidden') concepts

Information Filtering + LSI

Main idea

- map each document into some **'concepts'**
- map each term into some **'concepts'**

'Concept':~ a set of terms, with weights,
e.g. DBMS_concept:
"data" (0.8),
"system" (0.5),
"retrieval" (0.6)

Information Filtering + LSI

Pictorially: term-document matrix (BEFORE)

	'data'	'system'	'retrieval'	'lung'	'ear'
TR1	1	1	1		
TR2	1	1	1		
TR3				1	1
TR4				1	1

Information Filtering + LSI

Pictorially: **concept-document** matrix and...

	'DBMS-concept'	'medical-concept'
TR1	1	
TR2	1	
TR3		1
TR4		1

Information Filtering + LSI

... and **concept-term** matrix

	'DBMS-concept'	'medical-concept'
data	1	
system	1	
retrieval	1	
lung		1
ear		1

Information Filtering + LSI

Q: How to search, e.g., for ‘system’?

Information Filtering + LSI

A: find the corresponding concept(s); and the corresponding documents

	'DBMS-concept'	'medical-concept'
data	1	
system	1	
retrieval	1	
lung		1
ear		1

	'DBMS-concept'	'medical-concept'
TR1	1	
TR2	1	
TR3		1
TR4		1

Information Filtering + LSI

A: find the corresponding concept(s); and the corresponding documents

	'DBMS-concept'	'medical-concept'
data	1	
system	1	
retrieval	1	
lung		1
ear		1

	'DBMS-concept'	'medical-concept'
TR1	1	
TR2	1	
TR3		1
TR4		1

Information Filtering + LSI

Thus it works like an (automatically constructed) thesaurus.

We may retrieve documents that DON'T have the term 'system', but they contain almost everything else ('data', 'retrieval')

LSI - Discussion

- Great idea,
  - to derive ‘concepts’ from documents
  - to build a ‘statistical thesaurus’ automatically
  - to reduce dimensionality (down to few “concepts”)
- How exactly SVD works? (Details, next)

Singular Value Decomposition (SVD)
Motivation

Problem #1

Text - LSI uses SVD find “concepts”

Problem #2

Compression / dimensionality reduction

SVD - Motivation

Problem #1: text - LSI: find "concepts"

term / document	data	information	retrieval	brain	lung
CS-TR1	1	1	1	0	0
CS-TR2	2	2	2	0	0
CS-TR3	1	1	1	0	0
CS-TR4	5	5	5	0	0
MED-TR1	0	0	0	2	2
MED-TR2	0	0	0	3	3
MED-TR3	0	0	0	1	1

SVD - Motivation

Customer-product, for recommendation system:

$$\begin{array}{cc} & \begin{array}{ccccc} \text{bread} & \text{lettuce} & \text{tomatos} & \text{beef} & \text{chicken} \end{array} \\ \begin{array}{c} \text{vegetarians} \\ \\ \text{meat eaters} \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

SVD - Motivation

Problem #2:
Compress / reduce dimensionality

Problem - Specification

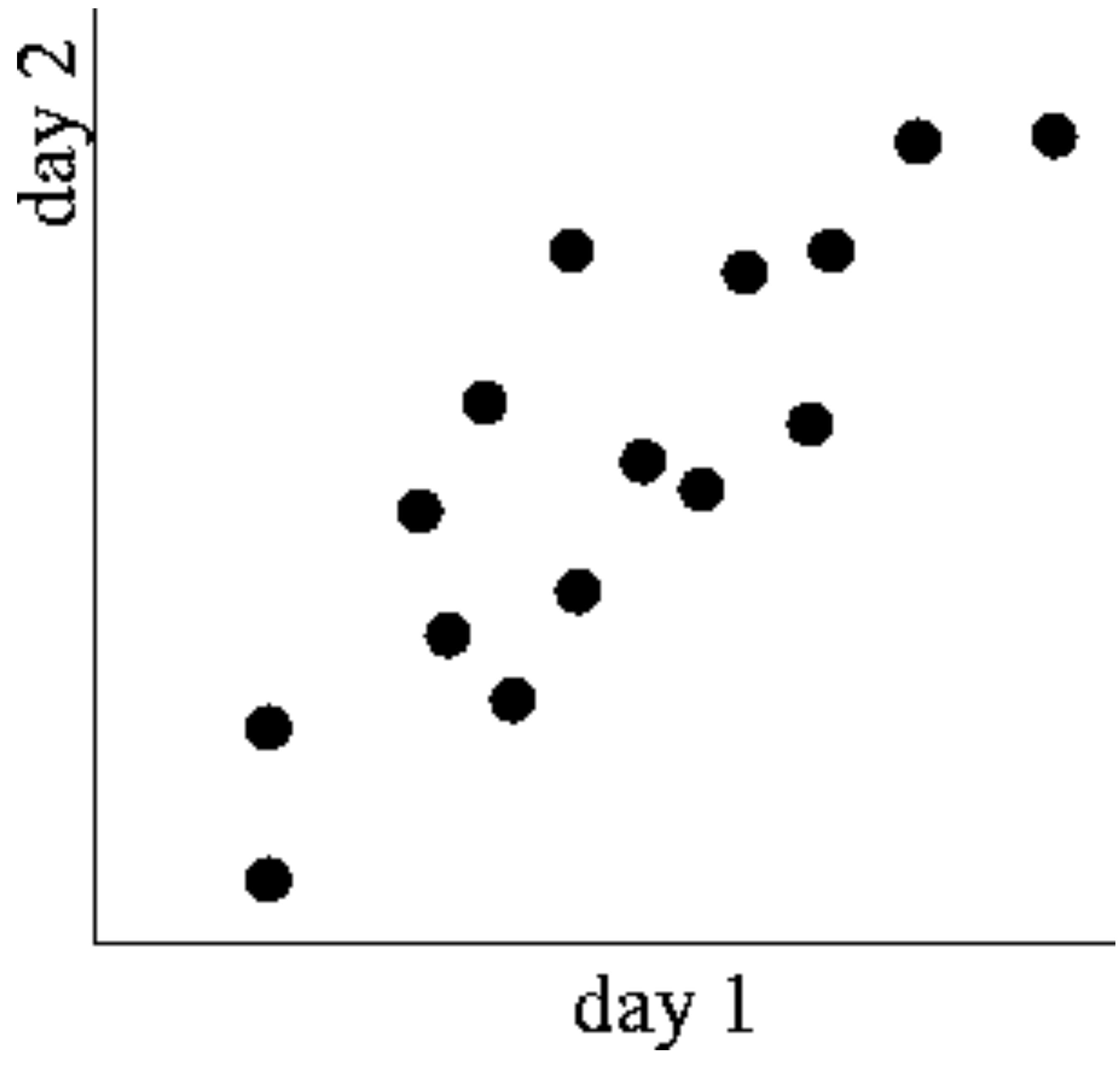
~10^6 rows; ~10^3 columns; no updates

Random access to any cell(s)

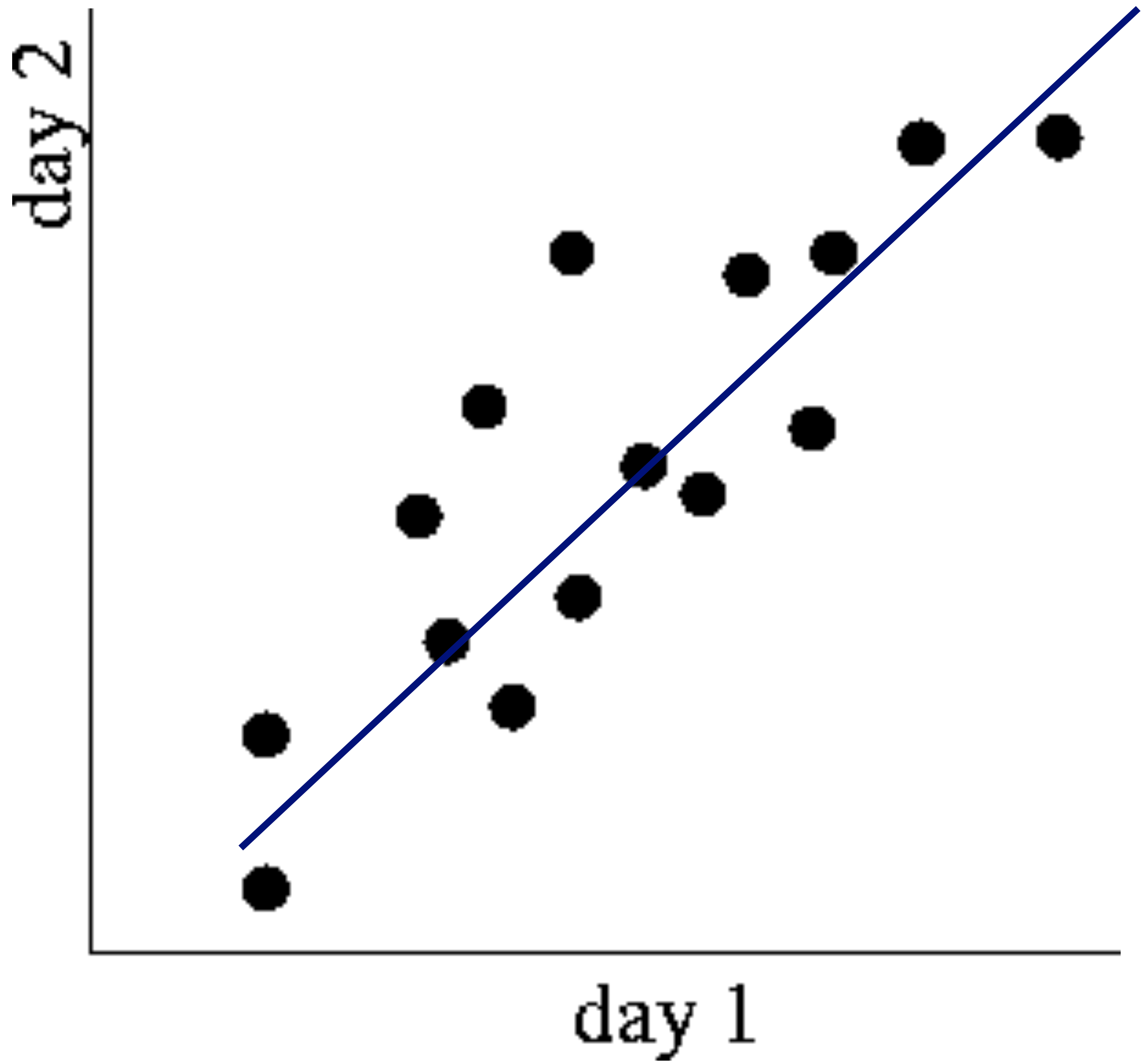
Small error: OK

day / customer	We 7/10/96	Th 7/11/96	Fr 7/12/96	Sa 7/13/96	Su 7/14/96
ABC Inc.	1	1	1	0	0
DEF Ltd.	2	2	2	0	0
GHI Inc.	1	1	1	0	0
KLM Co.	5	5	5	0	0
Smith	0	0	0	2	2
Johnson	0	0	0	3	3
Thompson	0	0	0	1	1

SVD - Motivation



SVD - Motivation



SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

3×2 $\qquad$ 2×1

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

3×2 $\quad$ 2×1 $\quad$ 3×1

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ \\ \\ \end{bmatrix}$$

$$3 \times 2 \qquad 2 \times 1 \qquad 3 \times 1$$

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \text{ x } \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}$$

3 x 2 $\quad$ 2 x 1 $\quad$ 3 x 1

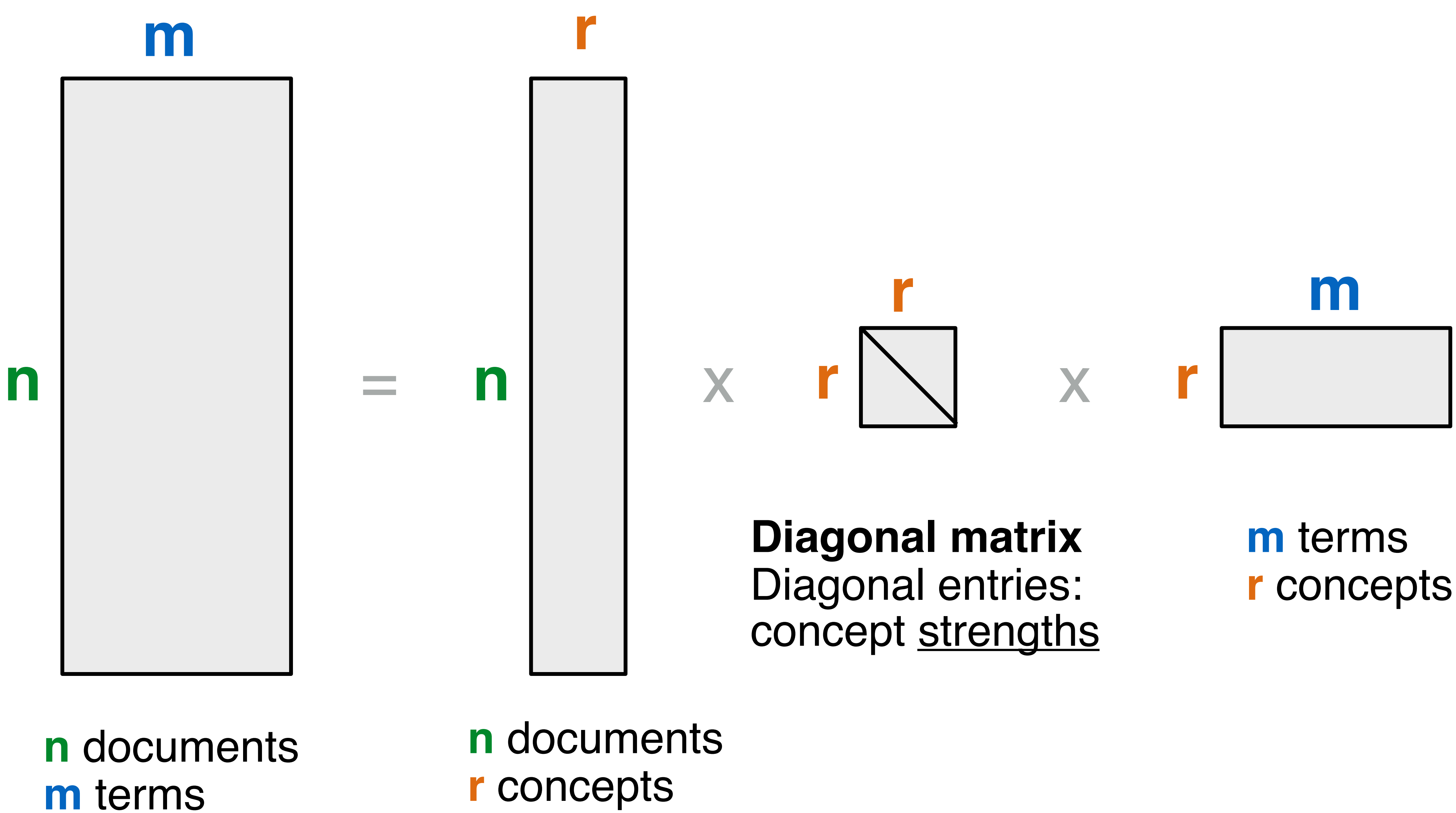
SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

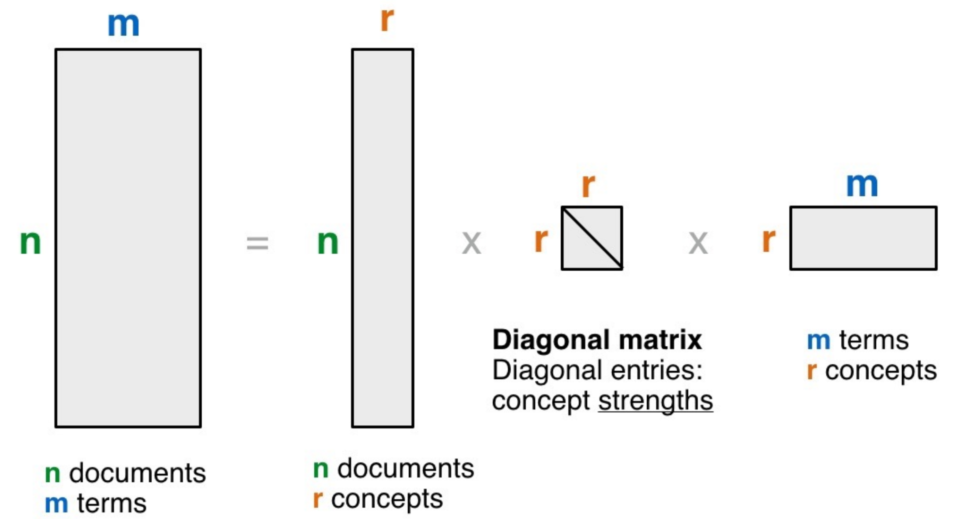
SVD Definition (in picture)

$$\boxed{A_{[n \times m]} = U_{[n \times r]} \, \Lambda_{[\, r \times r]} \, (V_{[m \times r]})^T}$$



SVD Definition (in words)

$$A_{[n \times m]} = U_{[n \times r]} \Lambda_{[r \times r]} (V_{[m \times r]})^T$$



A: n x m matrix

e.g., n documents, m terms

U: n x r matrix

e.g., n documents, r concepts

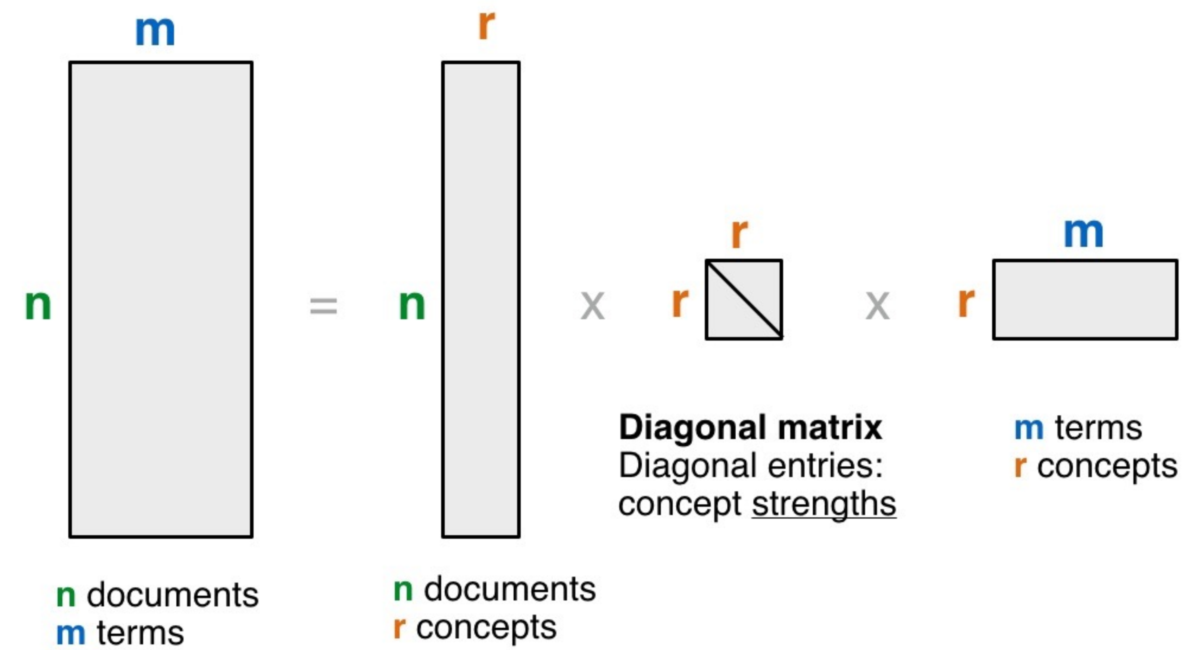
Λ: r x r diagonal matrix

r : rank of the matrix; strength of each ‘concept’

V: m x r matrix

e.g., m terms, r concepts

SVD - Properties



THEOREM [Press+92]:

always possible to decompose matrix $\mathbf{A}$ into

$$\mathbf{A} = \mathbf{U}\ \Lambda\ \mathbf{V}^T$$

$\mathbf{U}, \Lambda, \mathbf{V}$: **unique**, most of the time

$\mathbf{U}, \mathbf{V}$: column **orthonormal**

i.e., columns are unit vectors, and orthogonal to each other

$$\mathbf{U}^T\ \mathbf{U} = \mathbf{I}$$

$$\mathbf{V}^T\ \mathbf{V} = \mathbf{I}$$

(**I**: identity matrix)

Λ: **diagonal** matrix with non-negative diagonal entires, sorted in decreasing order

SVD - Example

$\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^{\mathrm{T}}$ - example:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \\ \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\, \Lambda\, \mathbf{V}^T$ - example:

CS-concept

MD-concept

$$
\begin{array}{c}
\\
\text{CS} \\
\\
\text{MD} \\
\\
\end{array}
\begin{array}{c}
\begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
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\end{bmatrix}
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$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\, \Lambda\, \mathbf{V}^T$ - example:

document-to-concept similarity matrix

CS-concept

MD-concept

$$
\begin{array}{c}
\\
\mathrm{CS} \\
\\
\\
\mathrm{MD} \\
\\
\end{array}
\begin{array}{c}
\begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
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\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\, \boldsymbol{\Lambda}\, \mathbf{V}^T$ - example:

‘strength’ of CS-concept

$$
\begin{array}{c c}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \text{CS} \\ \\ \\ \\ \text{MD} \\ \\ \\ \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
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\end{bmatrix}
\mathrm{x}
\begin{bmatrix}
9.64 & 0 \\
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\end{bmatrix}
\mathrm{x}
\begin{bmatrix}
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0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\,\Lambda\,\mathbf{V}^T$ - example:

term-to-concept similarity matrix

CS-concept

$$
\begin{array}{cc}
 & \begin{array}{ccccc}\text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung}\end{array} \\
\begin{array}{c}\text{CS}\\ \text{CS}\\ \text{CS}\\ \text{CS}\\ \text{MD}\\ \text{MD}\\ \text{MD}\end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
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0.36 & 0 \\
0.18 & 0 \\
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$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$ - example:

term-to-concept similarity matrix

$$\begin{array}{c|c} & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\ \begin{array}{c} \text{CS} \\ \\ \\ \text{MD} \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \text{x}$$

CS-concept

$$\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

$\mathbf{U}$: document-to-concept similarity matrix

$\mathbf{V}$: term-to-concept similarity matrix

Λ: diagonal elements: concept “strengths”

SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

Q: if $\mathbf{A}$ is the document-to-term matrix, what is the similarity matrix $\mathbf{A}^T \mathbf{A}$?

A:

Q: $\mathbf{A}\,\mathbf{A}^T$?

A:

SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

Q: if $\mathbf{A}$ is the document-to-term matrix, what is the similarity matrix $\mathbf{A}^{\mathsf{T}}\mathbf{A}$?

A: term-to-term ([m x m]) similarity matrix

Q: $\mathbf{A}\mathbf{A}^{\mathsf{T}}$?

A: document-to-document ([n x n]) similarity matrix

SVD properties

- $\mathbf{V}$ are the eigenvectors of the *covariance matrix* $\mathbf{A}^{\mathsf{T}}\mathbf{A}$

$$\mathbf{X}^{\mathsf{T}}\mathbf{X} = \left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right)^{\mathsf{T}}\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right) = \mathbf{V}\boldsymbol{\Sigma}^2\mathbf{V}^{\mathsf{T}}$$

- $\mathbf{U}$ are the eigenvectors of the *Gram (inner-product) matrix* $\mathbf{A}\mathbf{A}^{\mathsf{T}}$

$$\mathbf{X}\mathbf{X}^{\mathsf{T}} = \left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right)\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right)^{\mathsf{T}} = \mathbf{U}\boldsymbol{\Sigma}^2\mathbf{U}^{\mathsf{T}}$$

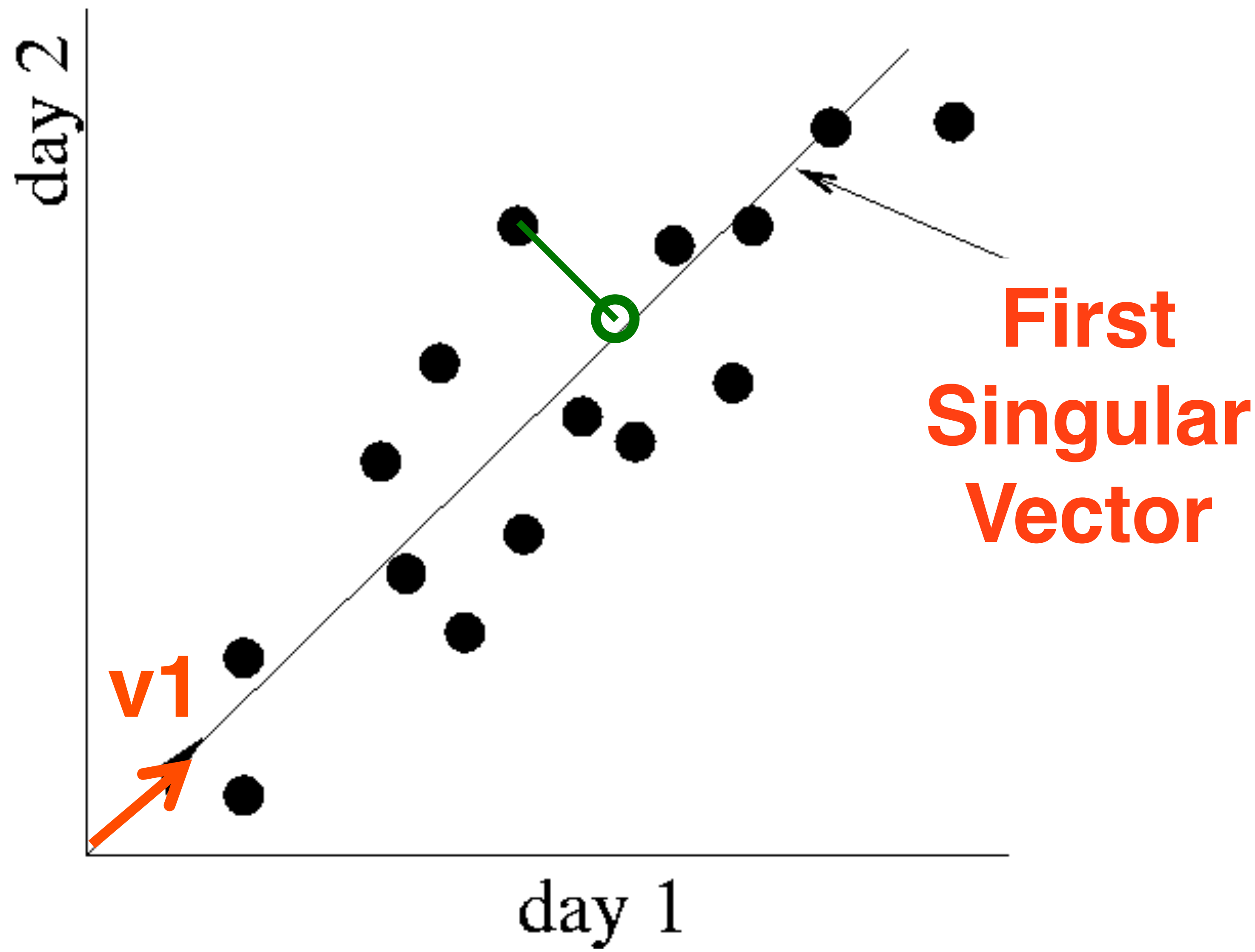
Thus, SVD is closely related to PCA, and can be numerically more stable.
For more info, see:

http://math.stackexchange.com/questions/3869/what-is-the-intuitive-relationship-between-svd-and-pca
Ian T. Jolliffe, *Principal Component Analysis* (2nd ed), Springer, 2002.
Gilbert Strang, *Linear Algebra and Its Applications* (4th ed), Brooks Cole, 2005.

SVD - Interpretation #2

Best axis to project on

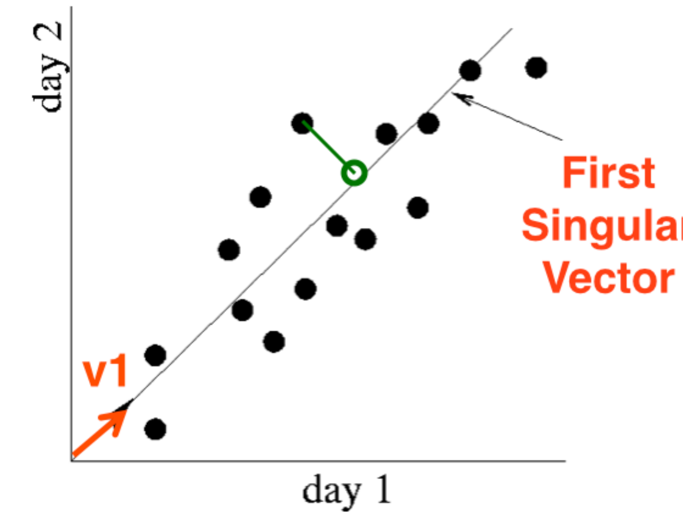
('best' = min sum of squares of projection errors)



min RMS error

Beautiful visualization explaining PCA:
http://setosa.io/ev/principal-component-analysis/

SVD - Interpretation #2



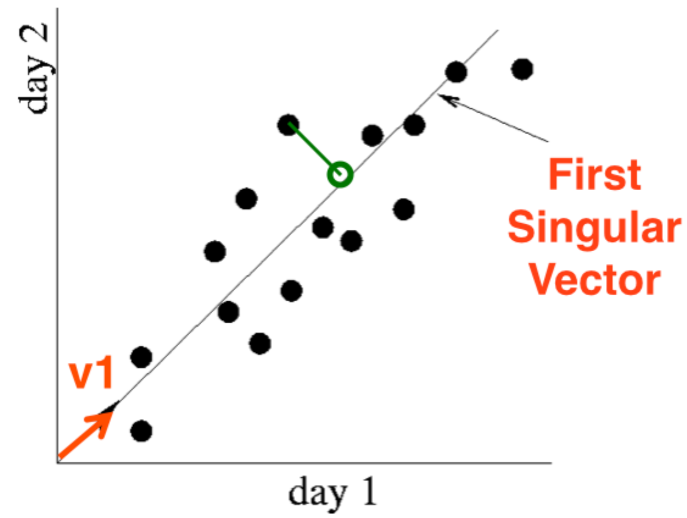
- $\mathbf{A} = \mathbf{U}\ \mathbf{\Lambda}\ \mathbf{V}^T$ - example:

variance ('spread') on the v1 axis

v1

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} \boxed{0.58 \quad 0.58 \quad 0.58 \quad 0 \quad 0} \\ 0 \quad 0 \quad 0 \quad 0.71 \quad 0.71 \end{bmatrix}$$

SVD - Interpretation #2



- $\mathbf{A} = \mathbf{U}\ \Lambda\ \mathbf{V}^T$ - example:
  - $\mathbf{U}\ \Lambda$ gives the **coordinates** of the points in the projection axis

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathbf{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathbf{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?

$$
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?
- A: set the smallest singular values to zero:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & \xcancel{5.29} \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

$$
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\sim
\begin{bmatrix}
0.18 & \cancel{0} \\
0.36 & \cancel{0} \\
0.18 & \cancel{0} \\
0.90 & \cancel{0} \\
0 & \cancel{0.53} \\
0 & \cancel{0.80} \\
0 & \cancel{0.27}
\end{bmatrix}
\mathbf{x}
\begin{bmatrix}
9.64 & 0 \\
0 & \cancel{0}
\end{bmatrix}
\mathbf{x}
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
\cancel{0} & \cancel{0} & \cancel{0} & \cancel{0.71} & \cancel{0.71}
\end{bmatrix}
$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 \\ 0.36 \\ 0.18 \\ 0.90 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 \end{bmatrix} \text{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

SVD - Interpretation #2

Exactly equivalent:

"spectral decomposition" of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathbf{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathbf{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

Exactly equivalent:

'spectral decomposition' of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix} \text{x} \begin{bmatrix} \lambda_1 & \emptyset \\ \emptyset & \lambda_2 \end{bmatrix} \text{x} \begin{bmatrix} \text{—} \; v_1 \; \text{—} \\ \text{—} \; v_2 \; \text{—} \end{bmatrix}$$

SVD - Interpretation #2

Exactly equivalent:

'spectral decomposition' of the matrix:

$$\begin{array}{cc} & \xleftarrow{\quad m \quad} \\ n\updownarrow & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \lambda_1 \; u_1 \; v^T{}_1 + \lambda_2 \; u_2 \; v^T{}_2 + \ldots$$

SVD - Interpretation #2

Exactly equivalent:

'spectral decomposition' of the matrix:

$$
\begin{array}{cc}
 & \overleftrightarrow{\quad m \quad} \\
n \updownarrow &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\overbrace{\lambda_1 \; \underbrace{u_1}_{n \times 1} \; \underbrace{v^T_1}_{1 \times m} + \lambda_2 \; u_2 \; v^T_2 + \ldots}^{r \text{ terms}}
$$

SVD - Interpretation #2

approximation / dim. reduction:

by keeping the first few terms (Q: how many?)

$$
n \left\{ \overbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}}^{m} \right. = \lambda_1 \quad u_1 \quad v^T_1 + \lambda_2 \quad u_2 \quad v^T_2 + \ldots
$$

assume: $\lambda_1 >= \lambda_2 >= \ldots$

SVD - Interpretation #2

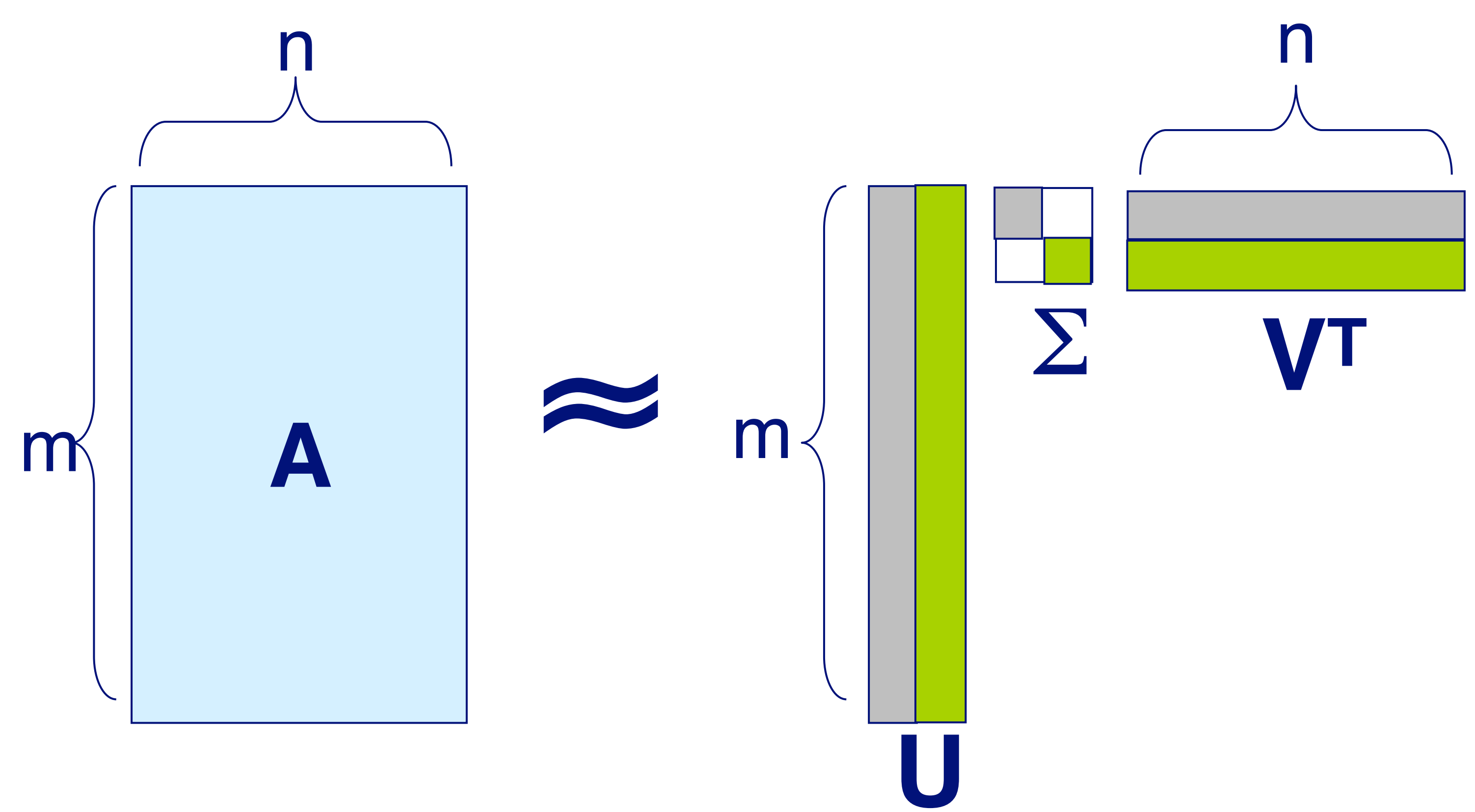
A (heuristic - [Fukunaga]): keep 80-90% of 'energy' (= sum of squares of λ_i 's)

$$
\begin{array}{cc}
 & \longleftarrow m \longrightarrow \\
n & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}
= \lambda_1 \quad u_1 \quad v^T_1 + \quad \lambda_2 \quad u_2 \quad v^T_2 + \ldots
$$

assume: $\lambda_1 >= \lambda_2 >= \ldots$

Pictorially: matrix form of SVD

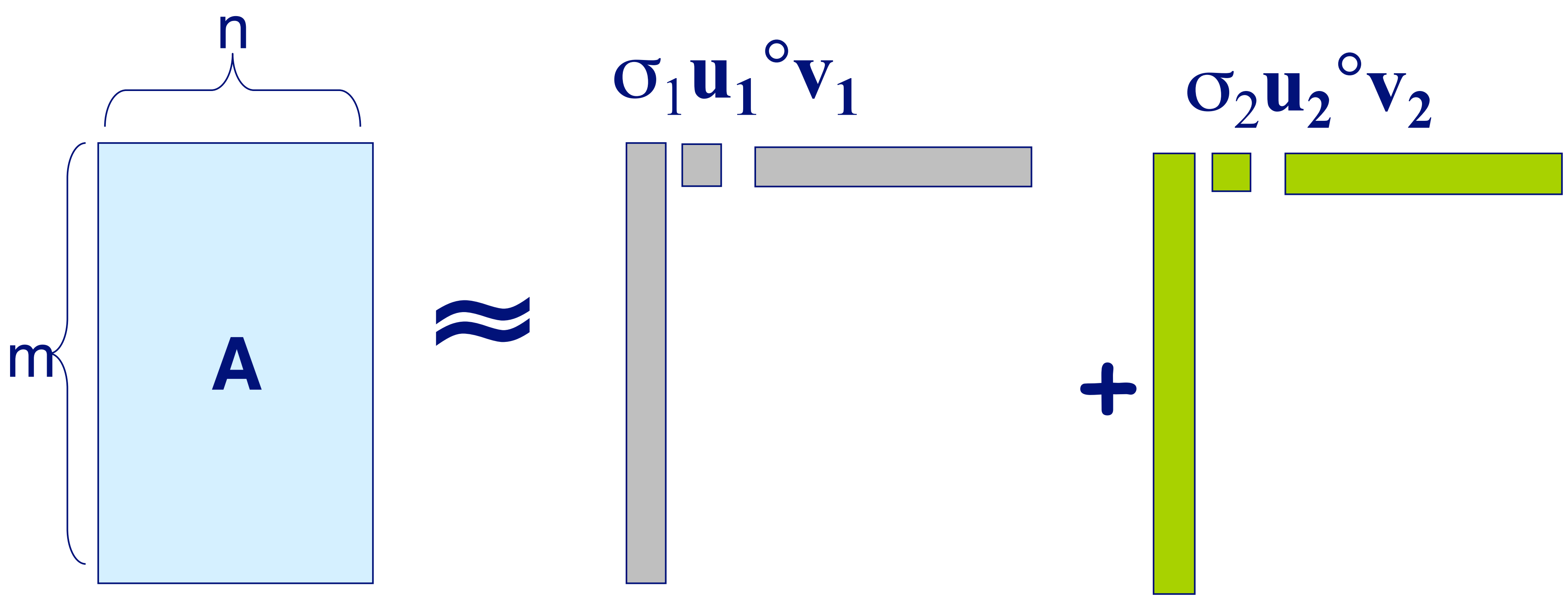
$$\mathbf{A} \approx \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_i \sigma_i \mathbf{u}_i \circ \mathbf{v}_i$$



– Best rank-k approximation in L2

Pictorially: Spectral form of SVD

$$\mathbf{A} \approx \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T = \sum_i \sigma_i \mathbf{u}_i \circ \mathbf{v}_i$$



– Best rank-k approximation in L2

SVD - Interpretation #3

- finds non-zero ‘blobs’ in a data matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #3

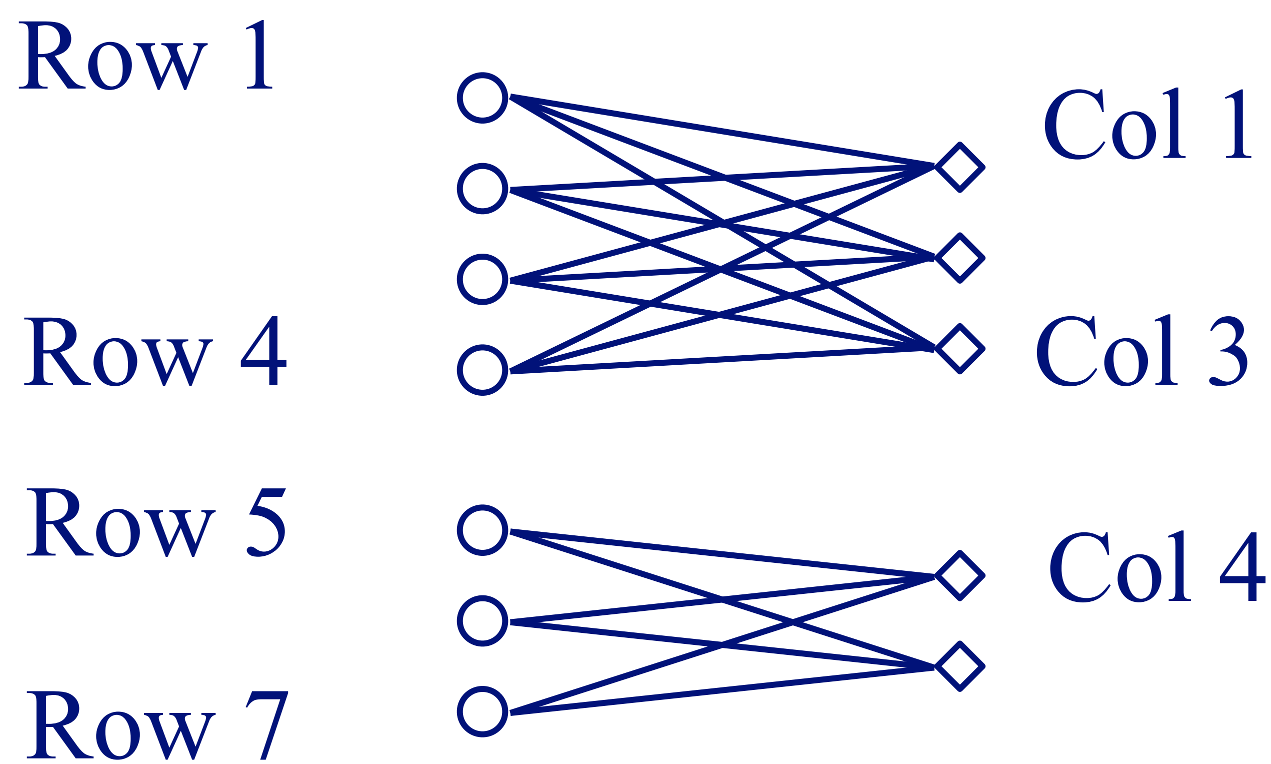
- finds non-zero 'blobs' in a data matrix

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #3

- finds non-zero ‘blobs’ in a data matrix =
- ‘communities’ (bi-partite cores, here)

$$\left[\begin{array}{ccc|cc}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
\hline
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{array}\right]$$



SVD algorithm

- Numerical Recipes in C (free)

SVD - Interpretation #3

- Drill: find the SVD, 'by inspection'!
- Q: rank = ??

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \\ \\ ?? \\ \\ \\ \end{bmatrix} \times \begin{bmatrix} \\ ?? \\ \\ \end{bmatrix} \times \begin{bmatrix} & & ?? & & \end{bmatrix}$$

SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/ cols)

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | \\ ?? & ?? \\ | & | \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} \text{---} & ?? & \text{---} \\ \text{---} & ?? & \text{---} \end{bmatrix}
$$

SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/ cols)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

orthogonal??

SVD - Interpretation #3

- column vectors: are orthogonal - but not unit vectors:

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 0 & 1/\mathrm{sqrt}(2) \\ 0 & 1/\mathrm{sqrt}(2) \end{bmatrix} \times \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \times \begin{bmatrix} 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 0 & 0 \\ 0 & 0 & 0 & 1/\mathrm{sqrt}(2) & 1/\mathrm{sqrt}(2) \end{bmatrix}
$$

SVD - Interpretation #3

- and the singular values are:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 0 & 1/\mathrm{sqrt}(2) \\ 0 & 1/\mathrm{sqrt}(2) \end{bmatrix} \mathrm{x} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \mathrm{x}$$

$$\begin{bmatrix} 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 0 & 0 \\ 0 & 0 & 0 & 1/\mathrm{sqrt}(2) & 1/\mathrm{sqrt}(2) \end{bmatrix}$$

SVD - Interpretation #3

- Q: How to check we are correct?

$$
\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 0 & 1/\mathrm{sqrt}(2) \\ 0 & 1/\mathrm{sqrt}(2) \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 0 & 0 \\ 0 & 0 & 0 & 1/\mathrm{sqrt}(2) & 1/\mathrm{sqrt}(2) \end{bmatrix}
$$

SVD - Interpretation #3

- A: SVD properties:
  - matrix product should give back matrix $\mathbf{A}$
  - matrix $\mathbf{U}$ should be column-orthonormal, i.e., columns should be unit vectors, orthogonal to each other
  - ditto for matrix $\mathbf{V}$
  - matrix Λ should be diagonal, with non-negative values

SVD - Complexity

O(n*m*m) or O(n*n*m) (whichever is less)

Faster version, if just want singular values

or if we want first k singular vectors

or if the matrix is sparse [Berry]

No need to write your own!

Available in most linear algebra packages (LINPACK, matlab, Splus/R, mathematica ...)

References

- Berry, Michael: http://www.cs.utk.edu/~lsi/
- Fukunaga, K. (1990). Introduction to Statistical Pattern Recognition, Academic Press.
- Press, W. H., S. A. Teukolsky, et al. (1992). Numerical Recipes in C, Cambridge University Press.

Case study - LSI

Q1: How to do queries with LSI?

Q2: multi-lingual IR (english query, on spanish text?)

Case study - LSI

Q1: How to do queries with LSI?

Problem: Eg., find documents with ‘data’

$$
\begin{array}{cc}
 & \begin{array}{ccccc}\text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung}\end{array} \\
\begin{array}{c} \\ \\ \text{CS} \\ \\ \\ \text{MD} \\ \\ \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}
=
\begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix}
\text{x}
\begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix}
\text{x}
\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}
$$

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into 'concept space' – how?

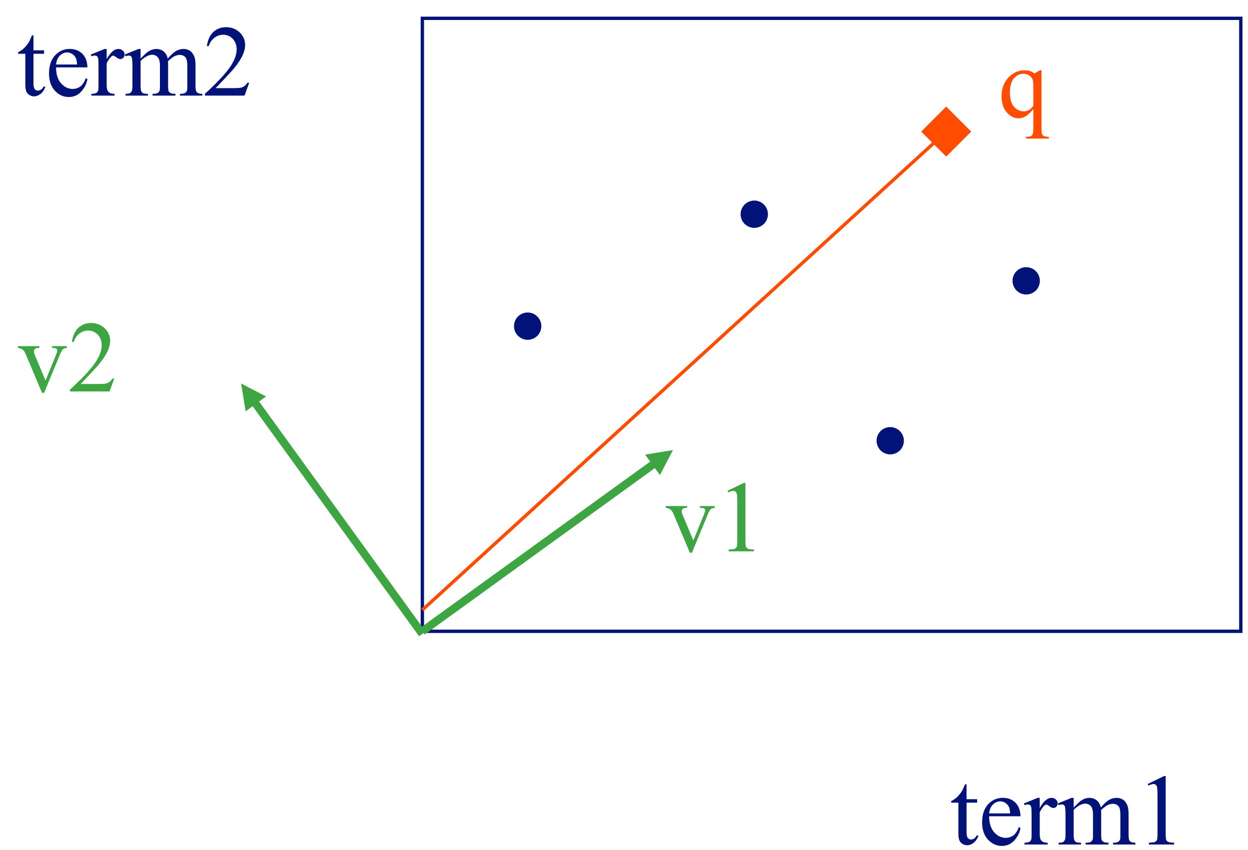
$$
\begin{array}{cc}
 & \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\
\begin{array}{c} \text{CS} \\ \text{CS} \\ \text{CS} \\ \text{CS} \\ \text{MD} \\ \text{MD} \\ \text{MD} \end{array} &
\begin{bmatrix}
1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into 'concept space' – how?

$$q = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}$$

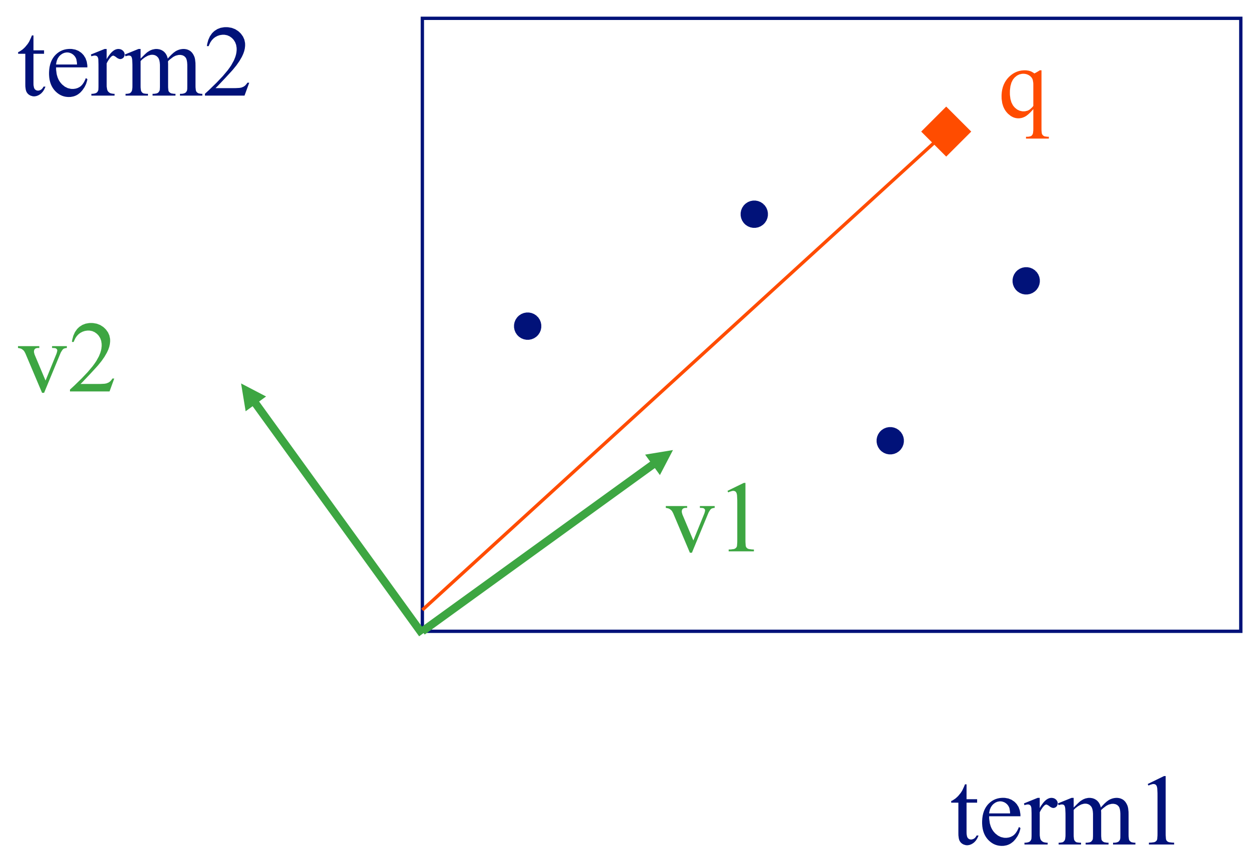


Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into 'concept space' – how?

$$
q = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \end{array}
$$
$$
q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}
$$



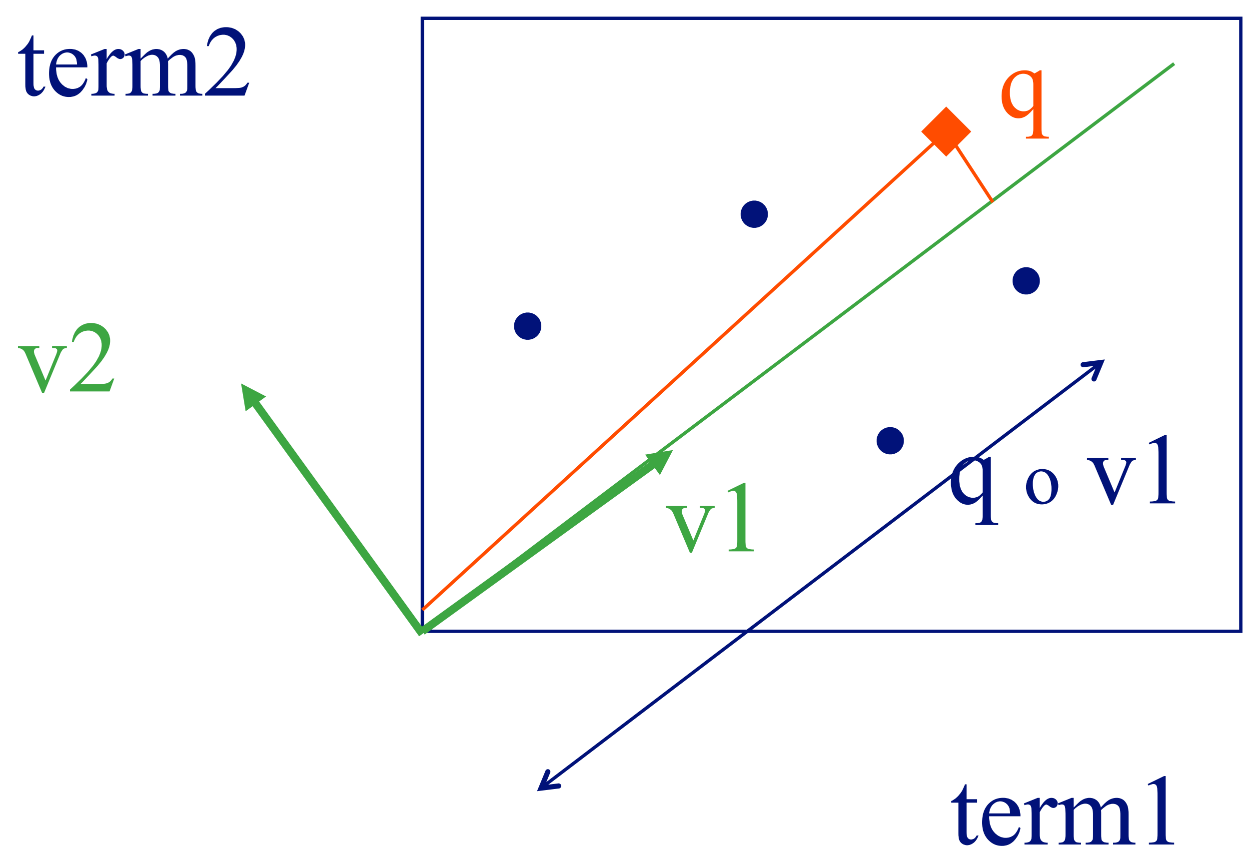
A: inner product
(cosine similarity)
with each 'concept' vector v_i

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into 'concept space' – how?

$$
q= \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \end{array}
$$
$$
q= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}
$$



A: inner product
(cosine similarity)
with each 'concept' vector v_i

Case study - LSI

compactly, we have:

$$q\,\mathbf{V} = q_{concept}$$

Eg:

$$q = \begin{array}{c} \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \underbrace{\begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.71 \\ 0 & 0.71 \end{bmatrix}}_{\text{term-to-concept similarities}} = \begin{array}{c} \text{CS-concept} \\ \begin{bmatrix} 0.58 & 0 \end{bmatrix} \end{array}$$

Case study - LSI

Drill: how would the document (‘information’, ‘retrieval’) be handled by LSI?

Case study - LSI

Drill: how would the document ('information', 'retrieval') be handled by LSI? A: SAME:

$d_{concept} = d \, \mathbf{V}$

Eg:

$$d = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \end{array}$$

$$d = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.71 \\ 0 & 0.71 \end{bmatrix} = \begin{bmatrix} 1.16 & 0 \end{bmatrix}$$

CS-concept

term-to-concept similarities

Case study - LSI

Observation: document ('information', 'retrieval') will be retrieved by query ('data'), although it does not contain 'data'!!

$$d = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline 0 & 1 & 1 & 0 & 0 \end{array} \dashrightarrow \begin{array}{cc} \text{CS-concept} & \\ \hline 1.16 & 0 \end{array}$$

$$q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \dashrightarrow \begin{bmatrix} 0.58 & 0 \end{bmatrix}$$

Case study - LSI

- Q1: How to do queries with LSI?
- Q2: multi-lingual IR (english query, on spanish text?)

Case study - LSI

- Problem:
  - given many documents, translated to both languages (eg., English and Spanish)
  - answer queries across languages

Case study - LSI

- Solution: ~ LSI

$$
\begin{array}{c}
 \\
\text{CS} \\
 \\
 \\
 \\
\text{MD} \\
 \\
\end{array}
\begin{array}{ccccc}
\text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
\end{array}
\quad
\begin{array}{cc}
\text{datos} & \text{informacion} \\
\end{array}
$$

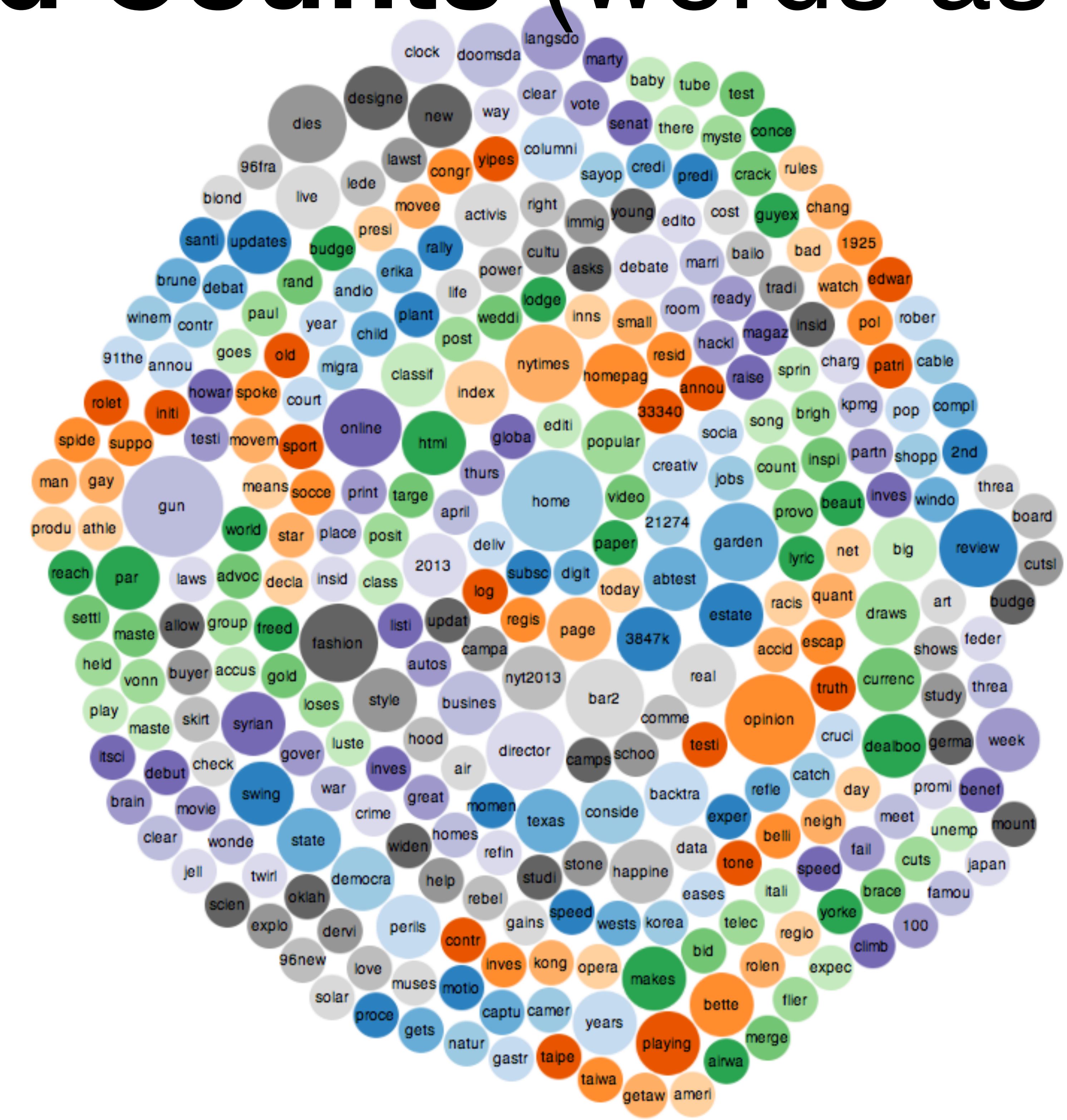
$$
\left[\begin{array}{ccccc:ccccc}
1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
2 & 2 & 2 & 0 & 0 & 1 & 2 & 2 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
5 & 5 & 5 & 0 & 0 & 5 & 5 & 4 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\
0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 2 & 3 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}\right]
$$

Switch Gear to
Text Visualization

Word/Tag Cloud (still popular?)

http://www.wordle.net

Word Counts (words as bubbles)



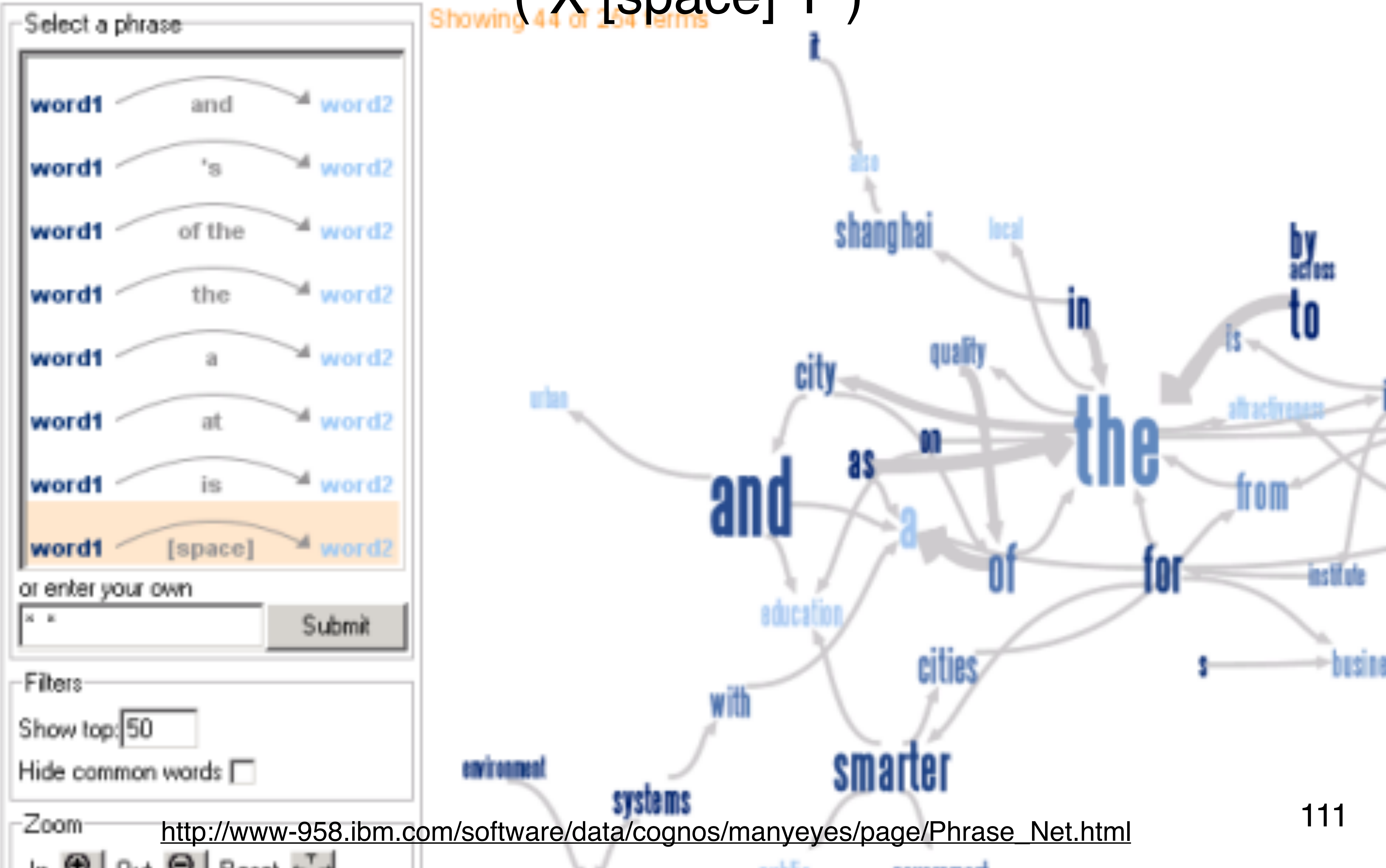
http://www.infocaptor.com/bubble-my-page

Word Tree

http://www.jasondavies.com/wordtree/

Phrase Net

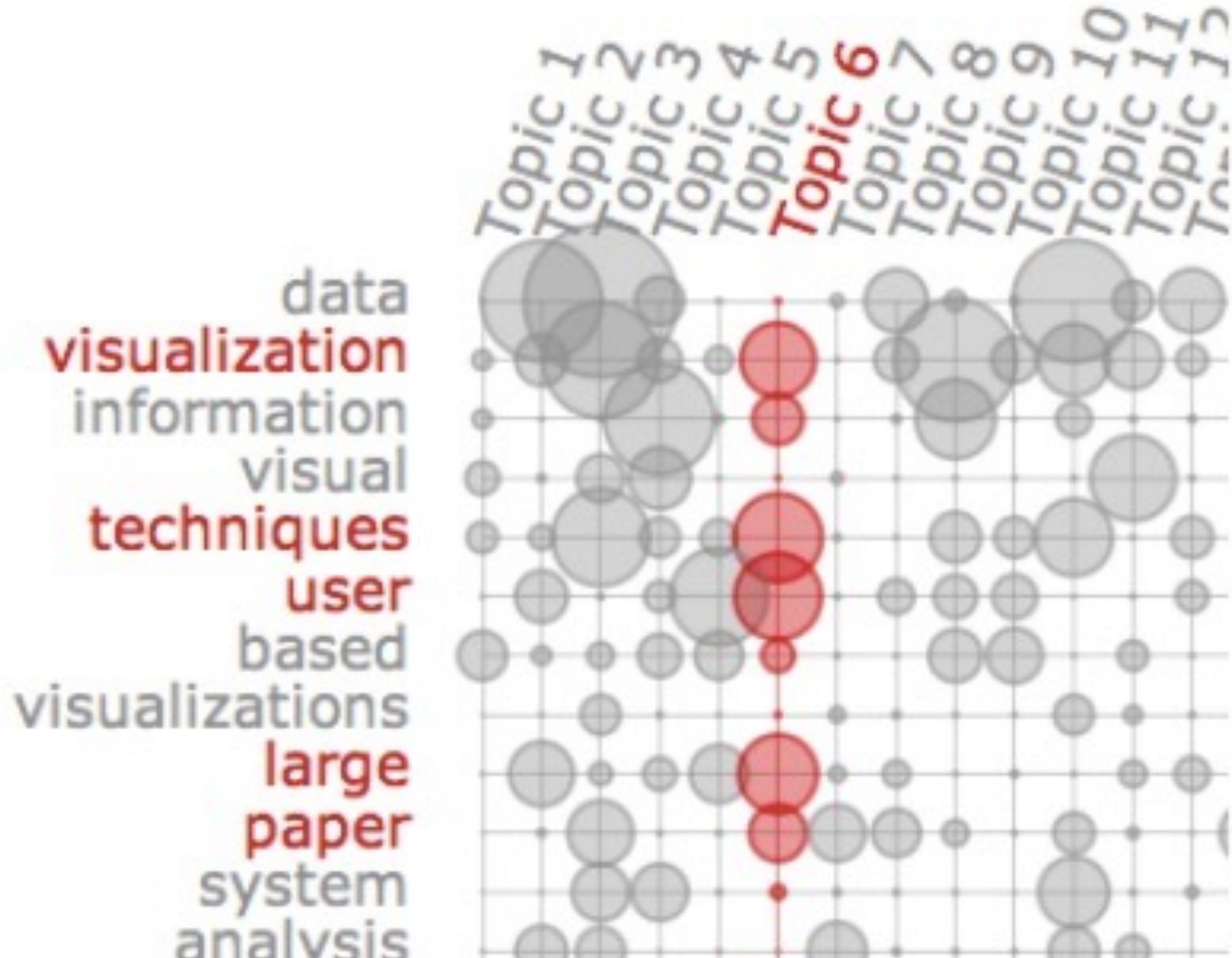
Visualize pairs of words satisfying a pattern
("X [space] Y")



http://www-958.ibm.com/software/data/cognos/manyeyes/page/Phrase_Net.html

Termite: Topic Model Visualization Analy

http://vis.stanford.edu/papers/termite



Termite: Topic Model Visualization

Analy

http://vis.stanford.edu/papers/termite