CSE 6242 / CX 4242

Text Analytics (Text Mining)

LSI (uses SVD), Visualization

Duen Horng (Polo) Chau
Georgia Tech

Some lectures are partly based on materials by Professors Guy Lebanon, Jeffrey Heer, John Stasko, Christos Faloutsos, Le Song

Singular Value Decomposition (SVD):
Motivation

Problem #1:

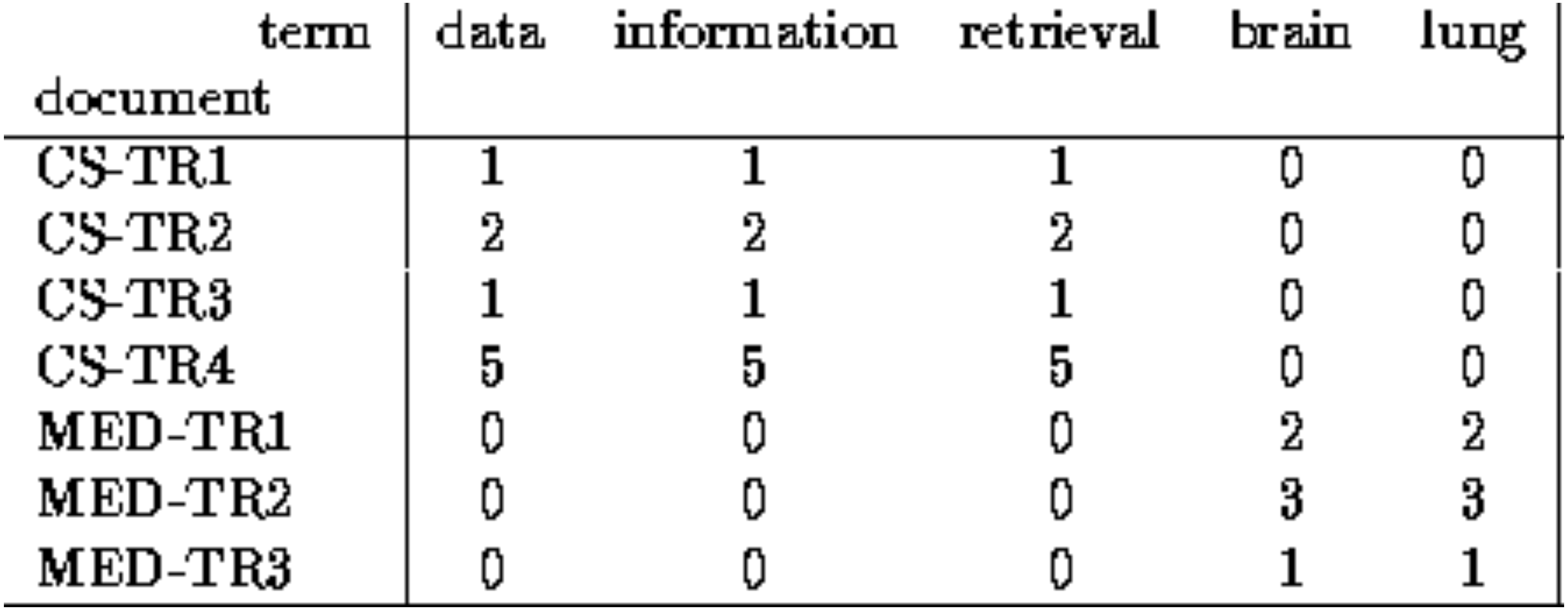
Text - LSI uses SVD find “concepts”

Problem #2:

Compression / dimensionality reduction

SVD - Motivation

Problem #1: text - LSI: find “concepts”



term document	data	information	retrieval	brain	lung
CS-TR1	1	1	1	0	0
CS-TR2	2	2	2	0	0
CS-TR3	1	1	1	0	0
CS-TR4	5	5	5	0	0
MED-TR1	0	0	0	2	2
MED-TR2	0	0	0	3	3
MED-TR3	0	0	0	1	1

SVD - Motivation

Customer-product, for recommendation system:

$$
\begin{array}{c|ccccc|}
 & \text{bread} & \text{lettuce} & \text{tomatos} & \text{beef} & \text{chicken} \\
\text{vegetarians} & 1 & 1 & 1 & 0 & 0 \\
 & 2 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 \\
\text{meat eaters} & 0 & 0 & 0 & 2 & 2 \\
 & 0 & 0 & 0 & 3 & 3 \\
 & 0 & 0 & 0 & 1 & 1
\end{array}
$$

SVD - Motivation

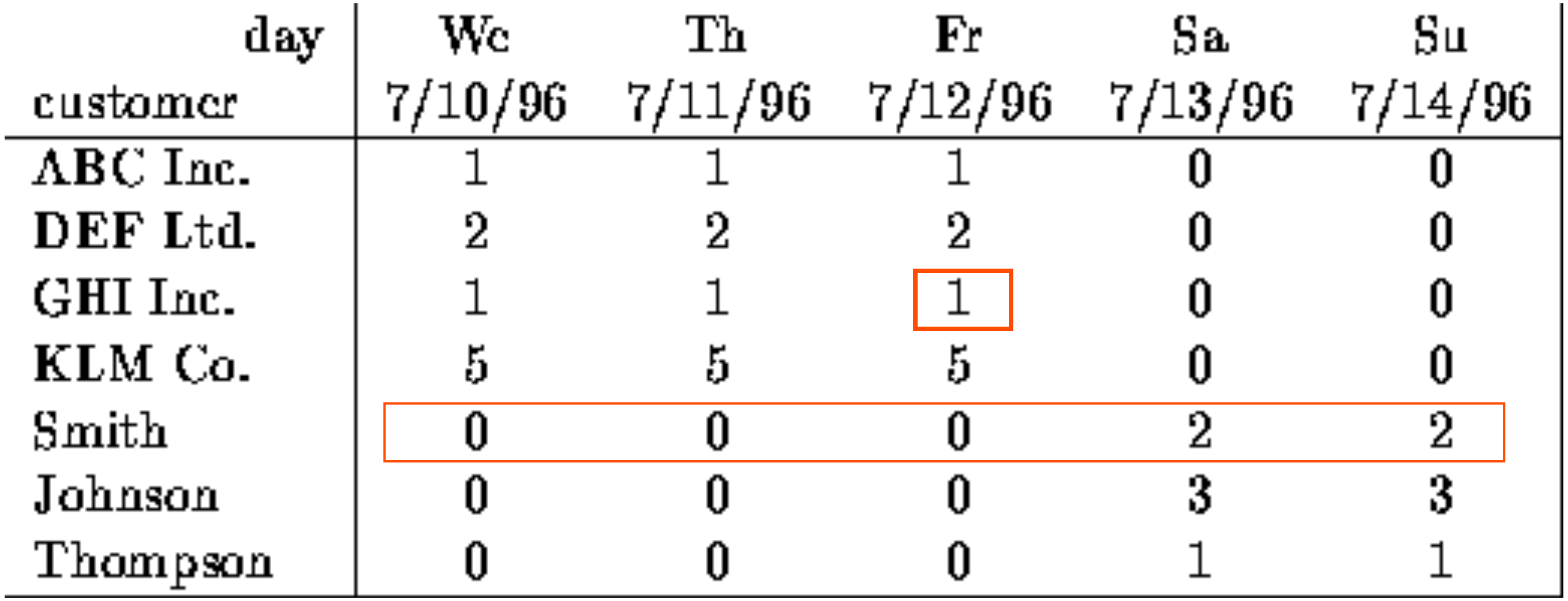
- problem #2: compress / reduce dimensionality

Problem - Specification

~10^6 rows; ~10^3 columns; no updates

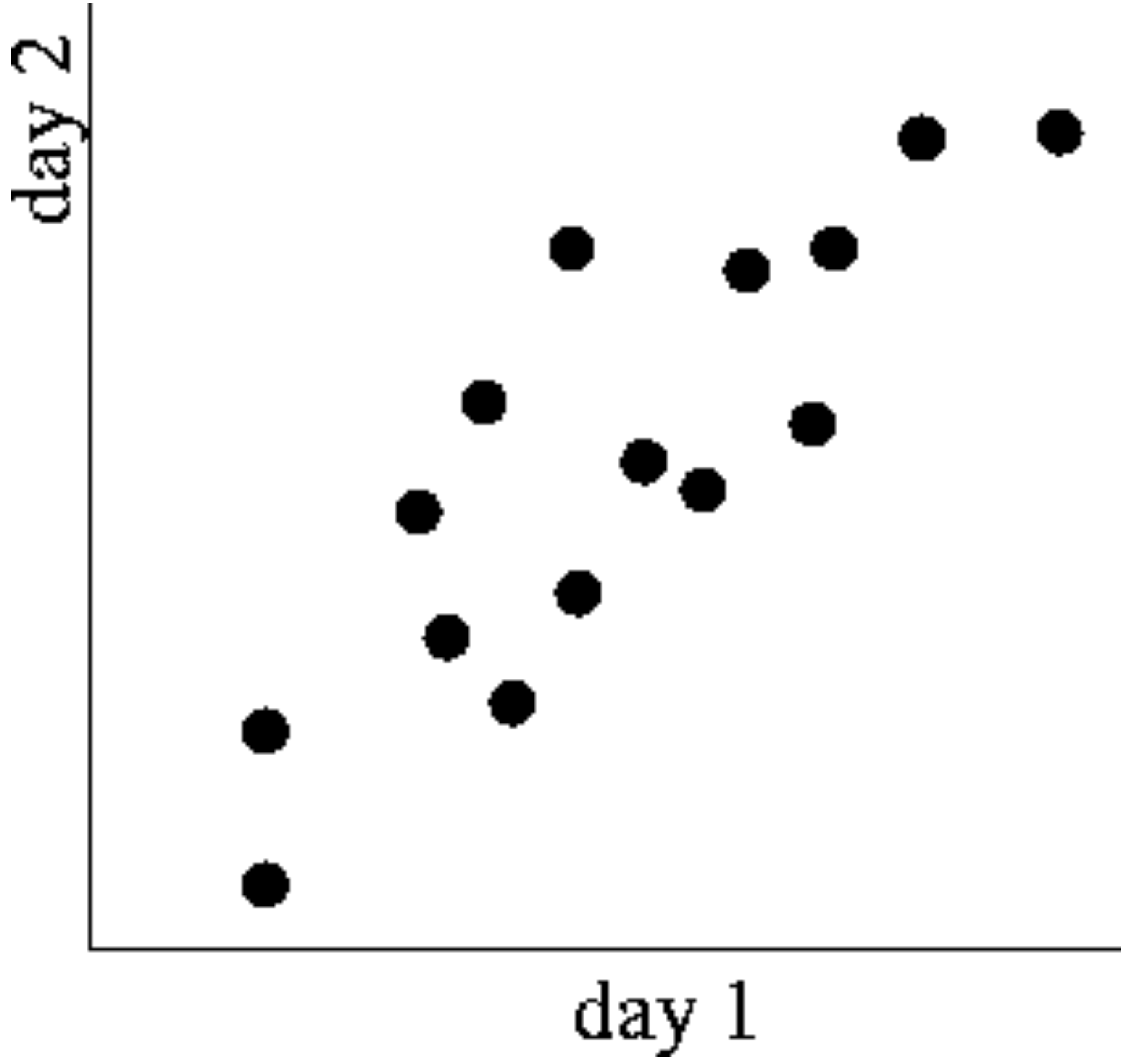
Random access to any cell(s)

Small error: OK

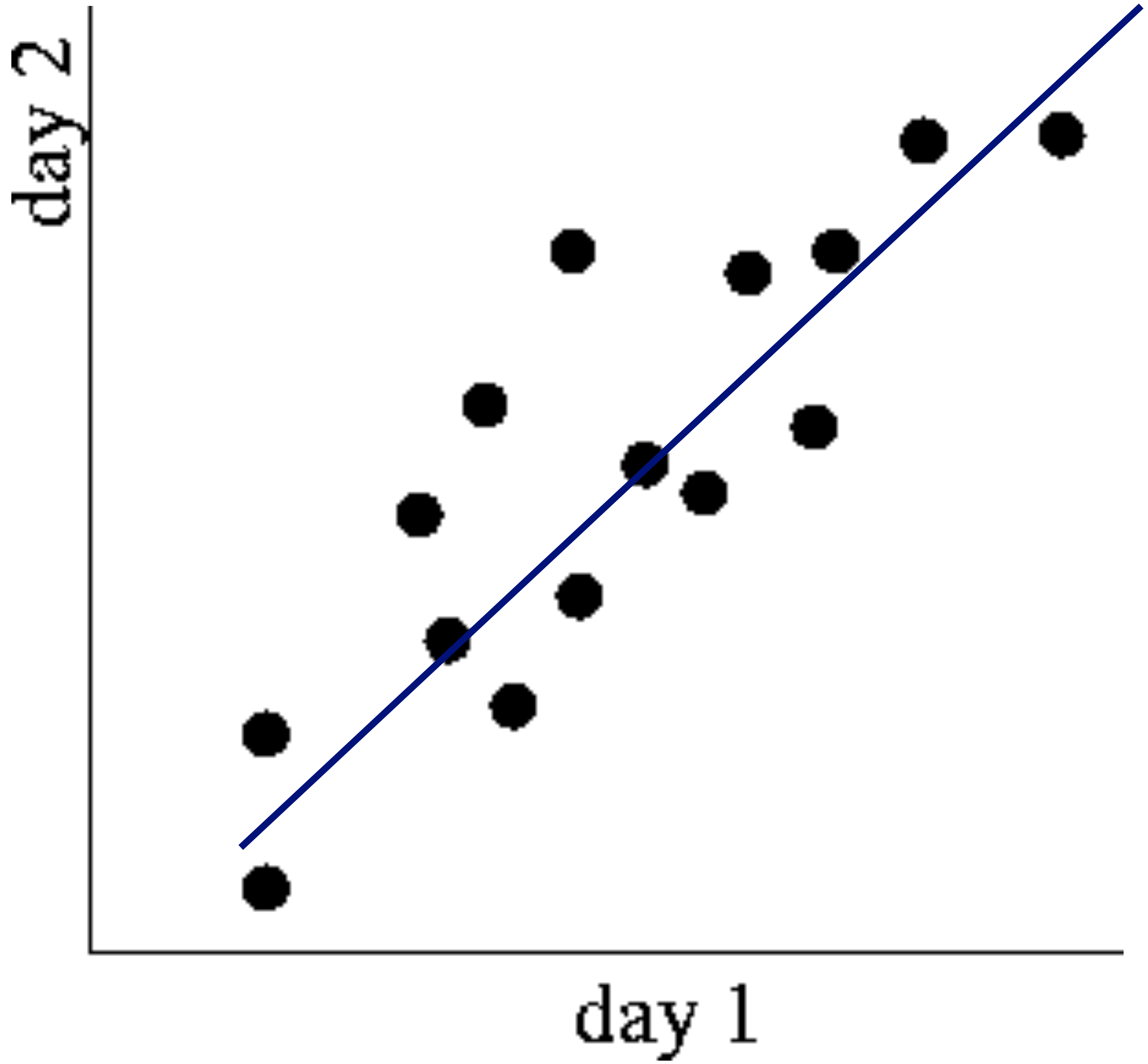


day	We	Th	Fr	Sa	Su
customer	7/10/96	7/11/96	7/12/96	7/13/96	7/14/96
ABC Inc.	1	1	1	0	0
DEF Ltd.	2	2	2	0	0
GHI Inc.	1	1	1	0	0
KLM Co.	5	5	5	0	0
Smith	0	0	0	2	2
Johnson	0	0	0	3	3
Thompson	0	0	0	1	1

SVD - Motivation



SVD - Motivation



SVD - Definition

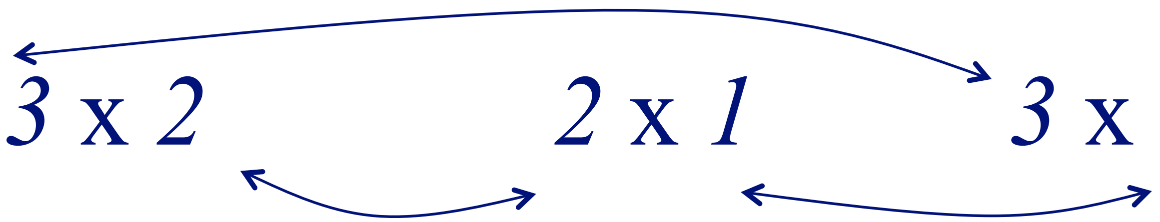
(reminder: matrix multiplication)

$$\underset{3 \text{ x } 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}} \text{ x } \underset{2 \text{ x } 1}{\begin{bmatrix} 1 \\ -1 \end{bmatrix}} = \begin{bmatrix} \quad \\ \\ \\ \end{bmatrix}$$

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$



SVD - Definition

(reminder: matrix multiplication)

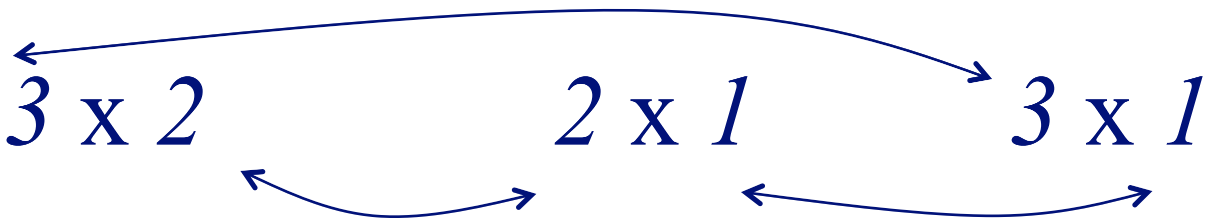
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ \\ \end{bmatrix}$$

$$3 \times 2 \qquad 2 \times 1 \qquad 3 \times 1$$

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ \end{bmatrix}$$



3 x 2 2 x 1 3 x 1

SVD - Definition

(reminder: matrix multiplication)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \text{x} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

SVD - Definition

$$\boxed{A_{[n \times m]} = U_{[n \times r]} \Lambda_{[r \times r]} (V_{[m \times r]})^T}$$

A: n x m matrix
e.g., n documents, m terms

U: n x r matrix
e.g., n documents, r concepts

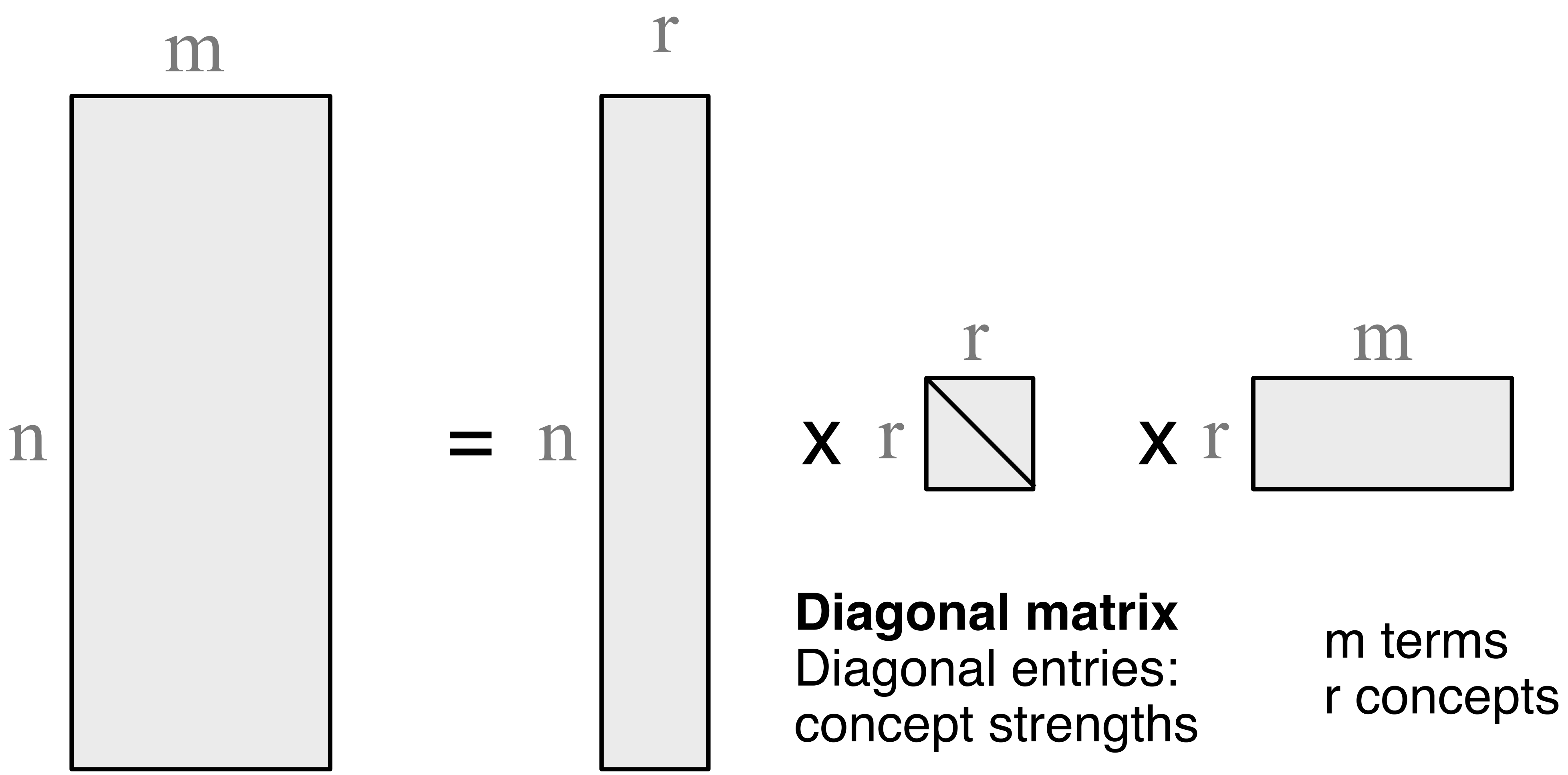
Λ: r x r diagonal matrix
r : rank of the matrix; strength of each ‘concept’

V: m x r matrix

e.g., m terms, r concepts

SVD - Definition

$$\mathbf{A}_{[n\ x\ m]} = \mathbf{U}_{[n\ x\ r]}\ \Lambda_{[\ r\ x\ r]}\ (\mathbf{V}_{[m\ x\ r]})^T$$



Diagonal matrix
Diagonal entries:
concept strengths

m terms
r concepts

n documents
m terms

n documents
r concepts

SVD - Properties

THEOREM [Press+92]:

always possible to decompose matrix $\mathbf{A}$ into

$\mathbf{A} = \mathbf{U} \, \mathbf{\Lambda} \, \mathbf{V}^T$

$\mathbf{U}$, $\mathbf{\Lambda}$, $\mathbf{V}$: **unique**, most of the time

$\mathbf{U}$, $\mathbf{V}$: column **orthonormal**

i.e., columns are unit vectors, and orthogonal to each other

$\mathbf{U}^T \mathbf{U} = \mathbf{I}$

$\mathbf{V}^T \mathbf{V} = \mathbf{I}$

($\mathbf{I}$: identity matrix)

$\mathbf{\Lambda}$: **diagonal** matrix with non-negative diagonal entires, sorted in decreasing order

SVD - Example

$\mathbf{A} = \mathbf{U}\, \Lambda\, \mathbf{V}^T$ - example:

$$
\begin{array}{c|ccccc|}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
\hline
 & 1 & 1 & 1 & 0 & 0 \\
\text{CS} & 2 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 \\
\hline
 & 0 & 0 & 0 & 2 & 2 \\
\text{MD} & 0 & 0 & 0 & 3 & 3 \\
 & 0 & 0 & 0 & 1 & 1 \\
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\text{x}
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\text{x}
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\, \mathbf{\Lambda}\, \mathbf{V}^T$ - example:

$$
\begin{array}{c}
\begin{array}{c|ccccc}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
\hline
\text{CS} & 1 & 1 & 1 & 0 & 0 \\
 & 2 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 \\
\text{MD} & 0 & 0 & 0 & 2 & 2 \\
 & 0 & 0 & 0 & 3 & 3 \\
 & 0 & 0 & 0 & 1 & 1
\end{array}
\end{array}
=
\begin{array}{cc}
\text{CS-concept} & \text{MD-concept} \\
\hline
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{array}
\ \text{x}\
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\ \text{x}\
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\,\Lambda\,\mathbf{V}^T$ - example:

doc-to-concept similarity matrix

$$
\begin{array}{c|ccccc}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
\hline
\text{CS} & 1 & 1 & 1 & 0 & 0 \\
 & 2 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 \\
\text{MD} & 0 & 0 & 0 & 2 & 2 \\
 & 0 & 0 & 0 & 3 & 3 \\
 & 0 & 0 & 0 & 1 & 1
\end{array}
=
\begin{array}{cc}
\text{CS-concept} & \text{MD-concept} \\
\hline
\boxed{0.18} & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{array}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

SVD - Example

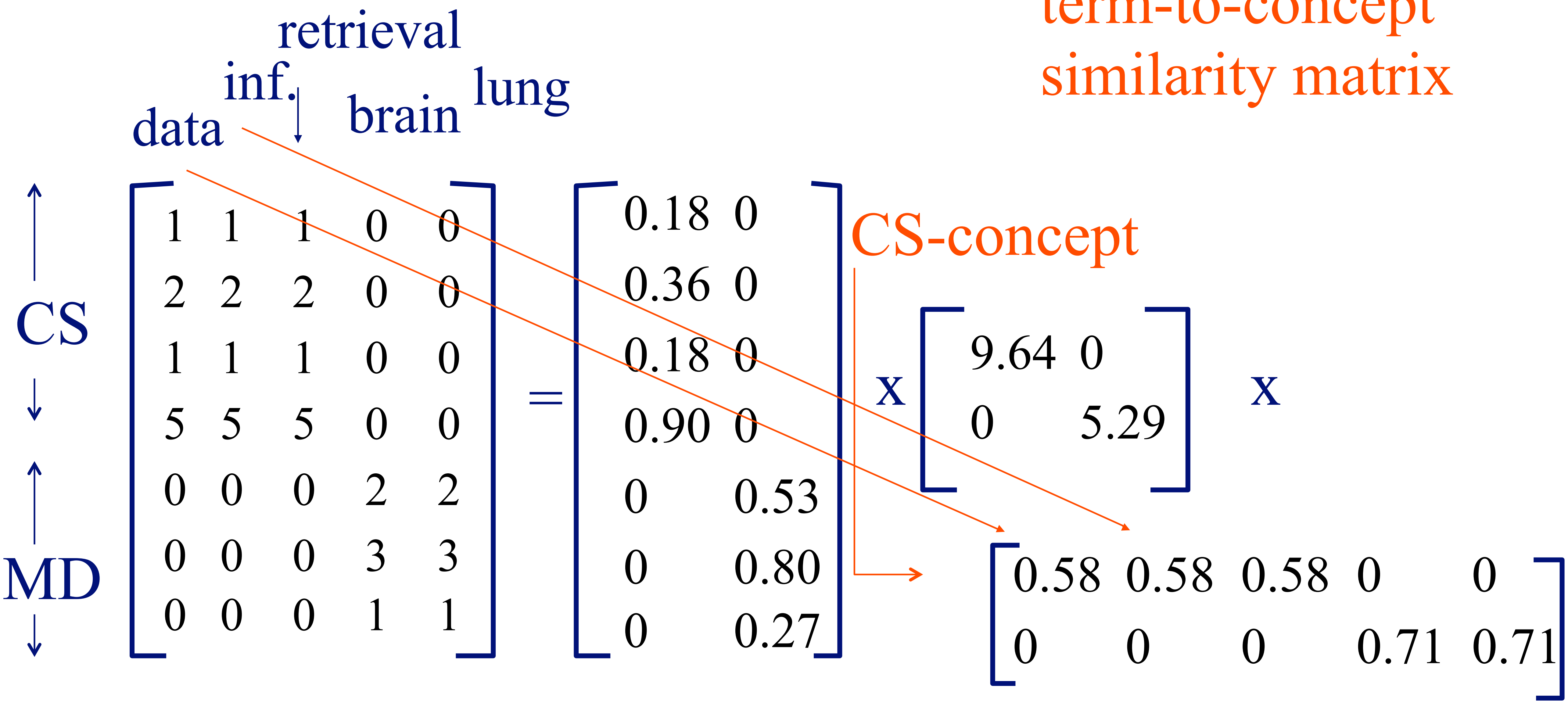
- $\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$ - example:

‘strength’ of CS-concept

$$\begin{array}{c|ccccc} & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline & 1 & 1 & 1 & 0 & 0 \\ \text{CS} & 2 & 2 & 2 & 0 & 0 \\ & 1 & 1 & 1 & 0 & 0 \\ & 5 & 5 & 5 & 0 & 0 \\ & 0 & 0 & 0 & 2 & 2 \\ \text{MD} & 0 & 0 & 0 & 3 & 3 \\ & 0 & 0 & 0 & 1 & 1 \end{array} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Example

- $\mathbf{A} = \mathbf{U}\ \Lambda\ \mathbf{V}^T$ - example:



$$\begin{array}{c|ccccc} & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline \text{CS} & 1 & 1 & 1 & 0 & 0 \\ & 2 & 2 & 2 & 0 & 0 \\ & 1 & 1 & 1 & 0 & 0 \\ & 5 & 5 & 5 & 0 & 0 \\ \text{MD} & 0 & 0 & 0 & 2 & 2 \\ & 0 & 0 & 0 & 3 & 3 \\ & 0 & 0 & 0 & 1 & 1 \end{array} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \times \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \times \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

term-to-concept similarity matrix

CS-concept

SVD - Example

- $\mathbf{A} = \mathbf{U}\,\mathbf{\Lambda}\,\mathbf{V}^T$ - example:

term-to-concept
similarity matrix

$$
\begin{array}{cccccc}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
\text{CS} & 1 & 1 & 1 & 0 & 0 \\
 & 2 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 \\
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 & 0 & 0 & 0 & 3 & 3 \\
 & 0 & 0 & 0 & 1 & 1
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\times
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\times
\begin{bmatrix}
\enclose{circle}{0.58} & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

CS-concept

SVD - Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

- **U**: document-to-concept similarity matrix
- **V**: term-to-concept similarity matrix
- Λ: diagonal elements: concept “strengths”

SVD – Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

Q: if $\mathbf{A}$ is the document-to-term matrix, what is $\mathbf{A}^T \mathbf{A}$?

A:

Q: $\mathbf{A} \mathbf{A}^T$?

A:

SVD – Interpretation #1

‘documents’, ‘terms’ and ‘concepts’:

Q: if $\mathbf{A}$ is the document-to-term matrix, what is $\mathbf{A}^T \mathbf{A}$?

A: term-to-term ([m x m]) similarity matrix

Q: $\mathbf{A} \mathbf{A}^T$?

A: document-to-document ([n x n]) similarity matrix

SVD properties

- **V** are the eigenvectors of the *covariance matrix* $\mathbf{A}^{\mathsf{T}}\mathbf{A}$

$$\mathbf{X}^{\mathsf{T}}\mathbf{X} = \left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right)^{\mathsf{T}}\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right) = \mathbf{V}\boldsymbol{\Sigma}^{2}\mathbf{V}^{\mathsf{T}}$$

- **U** are the eigenvectors of the *Gram (inner-product) matrix* $\mathbf{A}\mathbf{A}^{\mathsf{T}}$

$$\mathbf{X}\mathbf{X}^{\mathsf{T}} = \left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right)\left(\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathsf{T}}\right)^{\mathsf{T}} = \mathbf{U}\boldsymbol{\Sigma}^{2}\mathbf{U}^{\mathsf{T}}$$

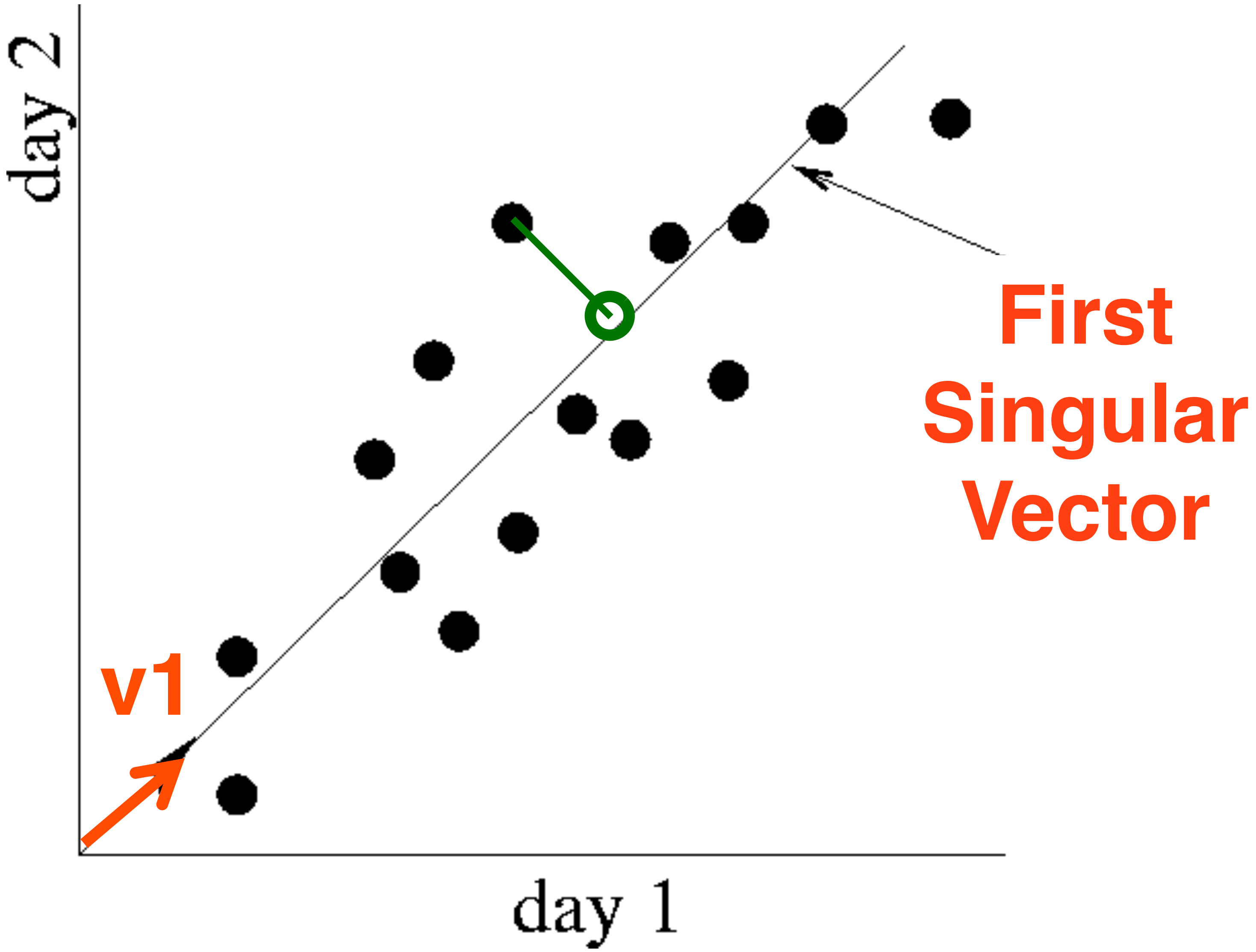
Thus, SVD is closely related to PCA, and can be numerically more stable.
For more info, see:

http://math.stackexchange.com/questions/3869/what-is-the-intuitive-relationship-between-svd-and-pca
Ian T. Jolliffe, *Principal Component Analysis* (2nd ed), Springer, 2002.
Gilbert Strang, *Linear Algebra and Its Applications* (4th ed), Brooks Cole, 2005.

SVD - Interpretation #2

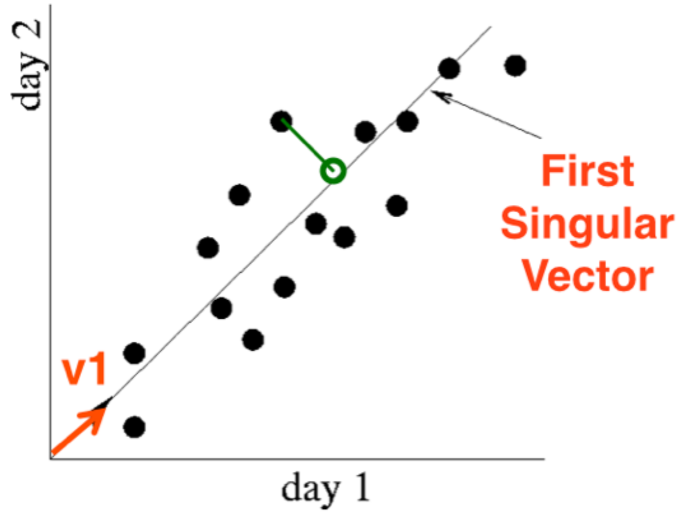
Best axis to project on

('best' = min sum of squares of projection errors)



min RMS error

SVD - Interpretation #2



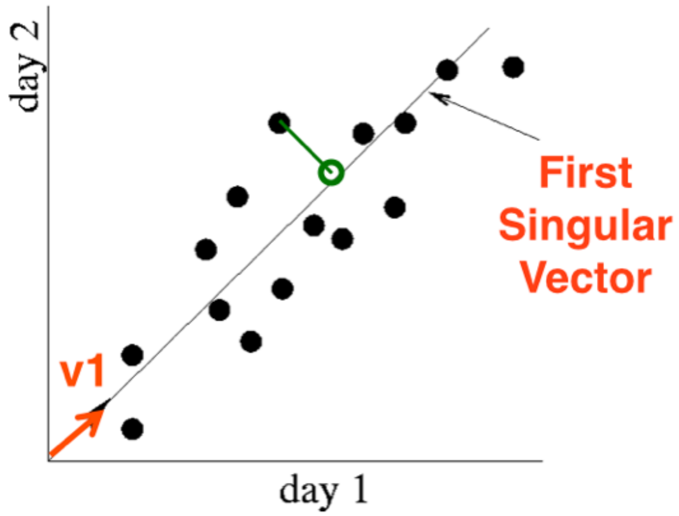
- **A** = **U Λ V**^{T} - example:

variance ('spread') on the v1 axis

v1

$$\begin{bmatrix}1&1&1&0&0\\2&2&2&0&0\\1&1&1&0&0\\5&5&5&0&0\\0&0&0&2&2\\0&0&0&3&3\\0&0&0&1&1\end{bmatrix} = \begin{bmatrix}0.18&0\\0.36&0\\0.18&0\\0.90&0\\0&0.53\\0&0.80\\0&0.27\end{bmatrix} \mathrm{x} \begin{bmatrix}\boxed{9.64}&0\\0&5.29\end{bmatrix} \mathrm{x} \begin{bmatrix}\boxed{0.58\quad 0.58\quad 0.58\quad 0\quad 0}\\0\quad 0\quad 0\quad 0.71\quad 0.71\end{bmatrix}$$

SVD - Interpretation #2



- $\mathbf{A} = \mathbf{U}\,\Lambda\,\mathbf{V}^T$ - example:
  - $\mathbf{U}\,\Lambda$ gives the **coordinates** of the points in the projection axis

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathrm{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathrm{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \text{x}$$

$$\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

- More details
- Q: how exactly is dim. reduction done?
- A: set the smallest singular values to zero:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & \xcancel{5.29} \end{bmatrix} \text{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix}1&1&1&0&0\\2&2&2&0&0\\1&1&1&0&0\\5&5&5&0&0\\0&0&0&2&2\\0&0&0&3&3\\0&0&0&1&1\end{bmatrix} \sim \begin{bmatrix}0.18&0\\0.36&0\\0.18&0\\0.90&0\\0&0.53\\0&0.80\\0&0.27\end{bmatrix} \text{ x } \begin{bmatrix}9.64&0\\0&\xcancel{0}\end{bmatrix} \text{ x } \begin{bmatrix}0.58&0.58&0.58&0&0\\0&0&0&0.71&0.71\end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 & \cancel{0} \\ 0.36 & \cancel{0} \\ 0.18 & \cancel{0} \\ 0.90 & \cancel{0} \\ 0 & \cancel{0.53} \\ 0 & \cancel{0.80} \\ 0 & \cancel{0.27} \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & \cancel{0} \end{bmatrix} \text{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0.71} & \cancel{0.71} \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 0.18 \\ 0.36 \\ 0.18 \\ 0.90 \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathrm{x} \begin{bmatrix} 9.64 \end{bmatrix} \mathrm{x}$$

$$\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \end{bmatrix}$$

SVD - Interpretation #2

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

SVD - Interpretation #2

Exactly equivalent:

“spectral decomposition” of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \text{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #2

Exactly equivalent:

‘spectral decomposition’ of the matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | \\ u_1 & u_2 \\ | & | \end{bmatrix} \text{x} \begin{bmatrix} \lambda_1 & \emptyset \\ \emptyset & \lambda_2 \end{bmatrix} \text{x} \begin{bmatrix} \text{---} & v_1 & \text{---} \\ \text{---} & v_2 & \text{---} \end{bmatrix}$$

SVD - Interpretation #2

Exactly equivalent:

‘spectral decomposition’ of the matrix:

$$n \updownarrow \overset{\longleftarrow \; m \; \longrightarrow}{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}} = \lambda_1 \quad u_1 \quad v^T{}_1 + \quad \lambda_2 \quad u_2 \quad v^T{}_2 + \ldots$$

SVD - Interpretation #2

Exactly equivalent:

‘spectral decomposition’ of the matrix:

$$\begin{array}{cc} & \xleftrightarrow{\quad m \quad} \\ n \updownarrow & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \overbrace{\lambda_1 \quad \underset{\text{n x 1}}{u_1} \quad \underset{\text{1 x m}}{v^T{}_1} + \quad \lambda_2 \quad u_2 \quad v^T{}_2 + \ldots}^{\text{r terms}}$$

SVD - Interpretation #2

approximation / dim. reduction:

by keeping the first few terms (Q: how many?)

$$n\updownarrow \overset{\xleftrightarrow{\quad m \quad}}{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}} = \lambda_1 \quad u_1 \quad v^T_1 + \quad \lambda_2 \quad u_2 \quad v^T_2 + \ldots$$

assume: $\lambda_1 >= \lambda_2 >= \ldots$

SVD - Interpretation #2

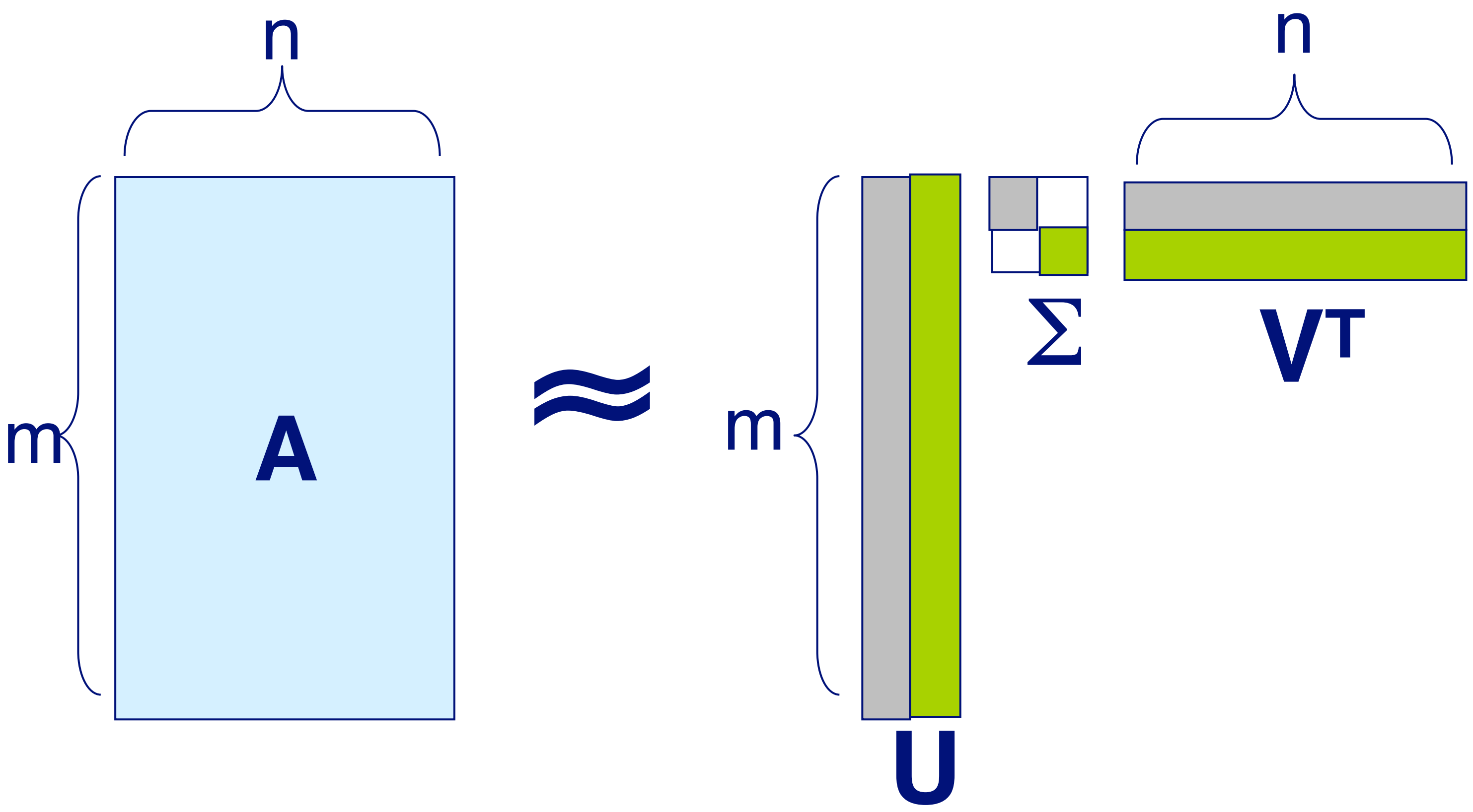
A (heuristic - [Fukunaga]): keep 80-90% of ‘energy’ (= sum of squares of λ_i ’s)

$$\begin{array}{cc} & \xleftrightarrow{\quad m \quad} \\ n \, \Big\updownarrow & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \lambda_1 \; u_1 \; v^T_1 + \lambda_2 \; u_2 \; v^T_2 + \ldots$$

assume: $\lambda_1 >= \lambda_2 >= \ldots$

Pictorially: matrix form of SVD

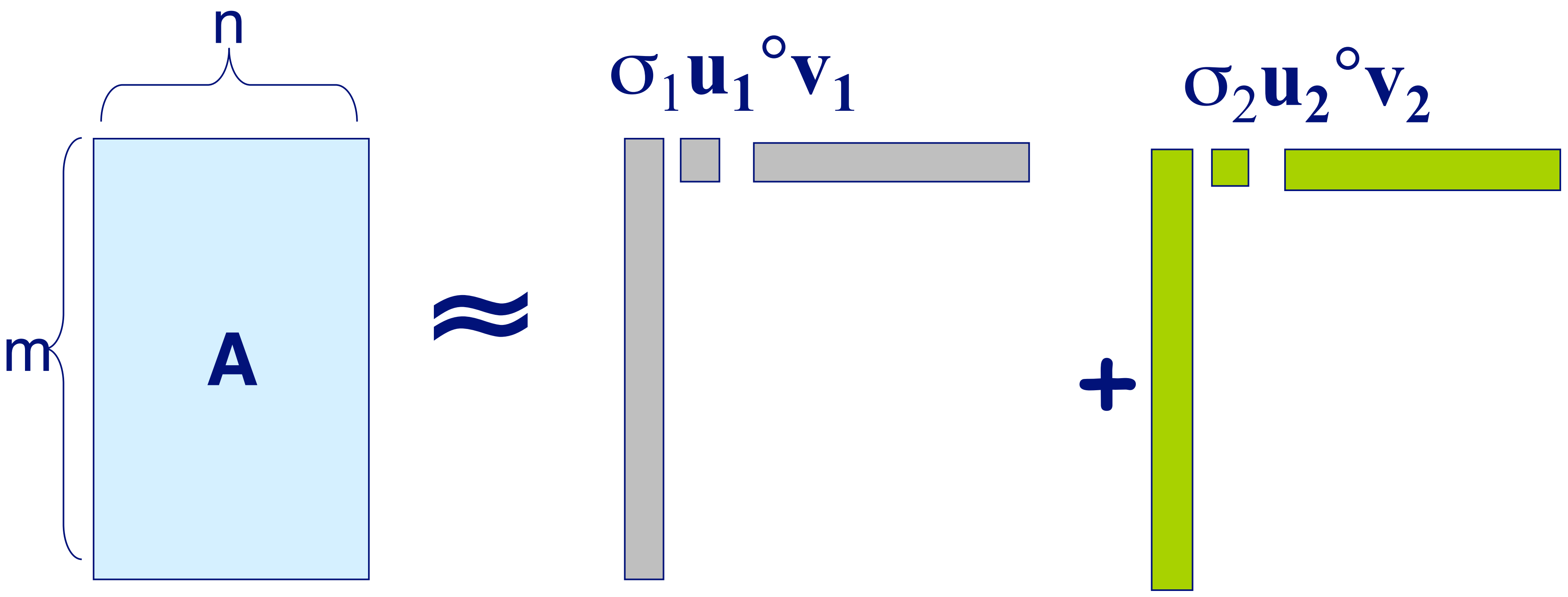
$$\mathbf{A} \approx \mathbf{U}\Sigma\mathbf{V}^T = \sum_i \sigma_i \mathbf{u}_i \circ \mathbf{v}_i$$



– Best rank-k approximation in L2

Pictorially: Spectral form of SVD

$$\mathbf{A} \approx \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \sum_i \sigma_i \mathbf{u}_i \circ \mathbf{v}_i$$



– Best rank-k approximation in L2

SVD - Interpretation #3

- finds non-zero ‘blobs’ in a data matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathrm{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathrm{x}$$

$$\begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #3

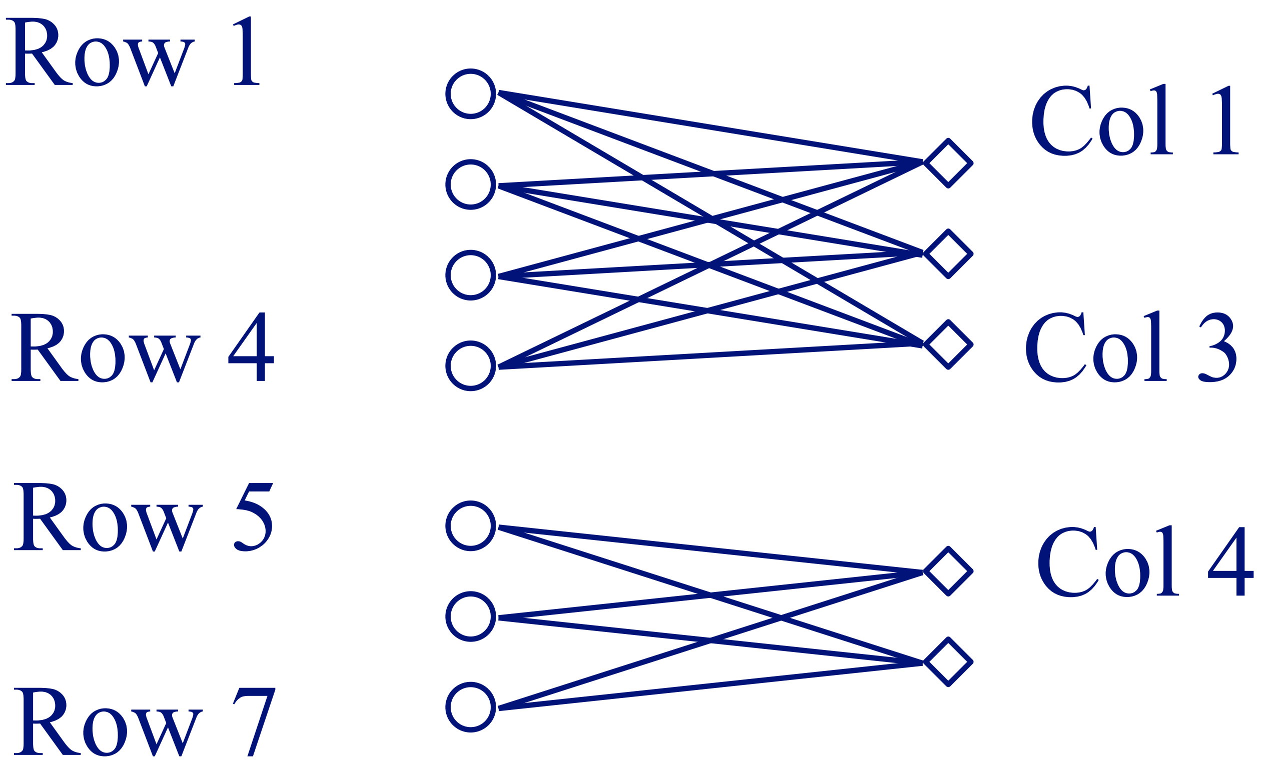
- finds non-zero ‘blobs’ in a data matrix

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \mathrm{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \mathrm{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

SVD - Interpretation #3

- finds non-zero ‘blobs’ in a data matrix =
- ‘communities’ (bi-partite cores, here)

$$\left[\begin{array}{ccc|cc} 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 5 & 5 & 5 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$



SVD algorithm

- Numerical Recipes in C (free)

SVD - Interpretation #3

- Drill: find the SVD, ‘by inspection’!
- Q: rank = ??

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \\ ?? \\ \\ \end{bmatrix} \mathrm{x} \begin{bmatrix} ?? \end{bmatrix} \mathrm{x}$$

$$\begin{bmatrix} ?? \end{bmatrix}$$

SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/cols)

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \vert & \vert \\ ?? & ?? \\ \vert & \vert \end{bmatrix} \text{ x } \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \text{ x } \begin{bmatrix} \text{———} ?? \text{———} \\ \text{———} ?? \text{———} \end{bmatrix}$$

SVD - Interpretation #3

- A: rank = 2 (2 linearly independent rows/cols)

$$\begin{bmatrix}1&1&1&0&0\\1&1&1&0&0\\1&1&1&0&0\\0&0&0&1&1\\0&0&0&1&1\end{bmatrix} = \begin{bmatrix}1&0\\1&0\\1&0\\0&1\\0&1\end{bmatrix} \; x \; \begin{bmatrix}??&0\\0&??\end{bmatrix} \; x \; \begin{bmatrix}1&1&1&0&0\\0&0&0&1&1\end{bmatrix}$$

orthogonal??

SVD - Interpretation #3

- column vectors: are orthogonal - but not unit vectors:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 1/\mathrm{sqrt}(3) & 0 \\ 0 & 1/\mathrm{sqrt}(2) \\ 0 & 1/\mathrm{sqrt}(2) \end{bmatrix} \mathrm{x} \begin{bmatrix} ?? & 0 \\ 0 & ?? \end{bmatrix} \mathrm{x}$$

$$\begin{bmatrix} 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 0 & 0 \\ 0 & 0 & 0 & 1/\mathrm{sqrt}(2) & 1/\mathrm{sqrt}(2) \end{bmatrix}$$

SVD - Interpretation #3

- and the singular values are:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \text{1/sqrt(3)} & 0 \\ \text{1/sqrt(3)} & 0 \\ \text{1/sqrt(3)} & 0 \\ 0 & \text{1/sqrt(2)} \\ 0 & \text{1/sqrt(2)} \end{bmatrix} \mathrm{x} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \mathrm{x}$$

$$\begin{bmatrix} \text{1/sqrt(3)} & \text{1/sqrt(3)} & \text{1/sqrt(3)} & 0 & 0 \\ 0 & 0 & 0 & \text{1/sqrt(2)} & \text{1/sqrt(2)} \end{bmatrix}$$

SVD - Interpretation #3

- Q: How to check we are correct?

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \text{1/sqrt(3)} & 0 \\ \text{1/sqrt(3)} & 0 \\ \text{1/sqrt(3)} & 0 \\ 0 & \text{1/sqrt(2)} \\ 0 & \text{1/sqrt(2)} \end{bmatrix} \text{x} \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \text{x}$$

$$\begin{bmatrix} \text{1/sqrt(3)} & \text{1/sqrt(3)} & \text{1/sqrt(3)} & 0 & 0 \\ 0 & 0 & 0 & \text{1/sqrt(2)} & \text{1/sqrt(2)} \end{bmatrix}$$

SVD - Interpretation #3

- A: SVD properties:
  - matrix product should give back matrix $\mathbf{A}$
  - matrix $\mathbf{U}$ should be column-orthonormal, i.e., columns should be unit vectors, orthogonal to each other
  - ditto for matrix $\mathbf{V}$
  - matrix Λ should be diagonal, with non-negative values

SVD - Complexity

O(n*m*m) or O(n*n*m) (whichever is less)

Faster version, if just want singular values
or if we want first k singular vectors
or if the matrix is sparse [Berry]

No need to write your own!
Available in most linear algebra packages (LINPACK, matlab, Splus/R, mathematica ...)

References

- Berry, Michael: http://www.cs.utk.edu/~lsi/
- Fukunaga, K. (1990). Introduction to Statistical Pattern Recognition, Academic Press.
- Press, W. H., S. A. Teukolsky, et al. (1992). Numerical Recipes in C, Cambridge University Press.

Case study - LSI

Q1: How to do queries with LSI?

Q2: multi-lingual IR (english query, on spanish text?)

Case study - LSI

Q1: How to do queries with LSI?

Problem: Eg., find documents with ‘data’

$$\begin{array}{c|ccccc} & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline \text{CS} & 1 & 1 & 1 & 0 & 0 \\ & 2 & 2 & 2 & 0 & 0 \\ & 1 & 1 & 1 & 0 & 0 \\ & 5 & 5 & 5 & 0 & 0 \\ \text{MD} & 0 & 0 & 0 & 2 & 2 \\ & 0 & 0 & 0 & 3 & 3 \\ & 0 & 0 & 0 & 1 & 1 \end{array} = \begin{bmatrix} 0.18 & 0 \\ 0.36 & 0 \\ 0.18 & 0 \\ 0.90 & 0 \\ 0 & 0.53 \\ 0 & 0.80 \\ 0 & 0.27 \end{bmatrix} \text{x} \begin{bmatrix} 9.64 & 0 \\ 0 & 5.29 \end{bmatrix} \text{x} \begin{bmatrix} 0.58 & 0.58 & 0.58 & 0 & 0 \\ 0 & 0 & 0 & 0.71 & 0.71 \end{bmatrix}$$

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

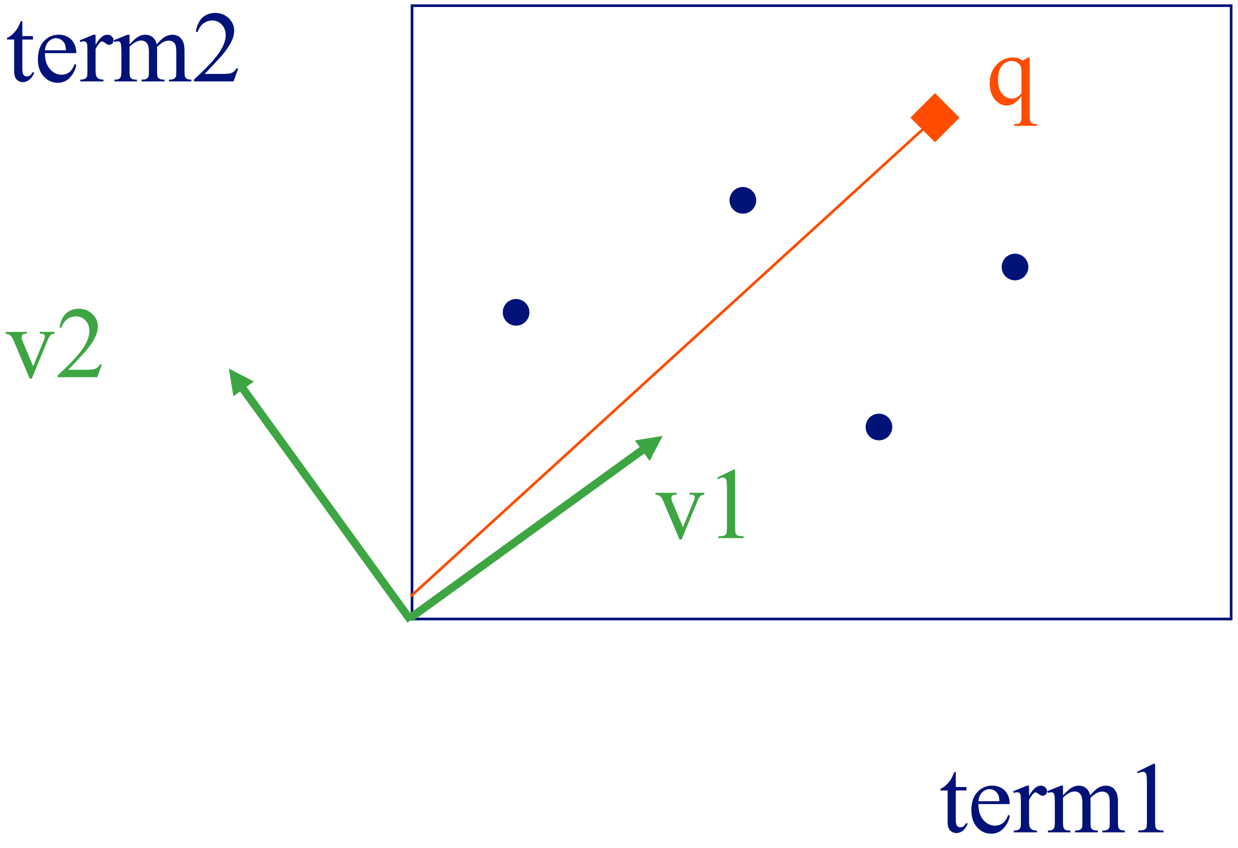
$$
\begin{array}{c|ccccc|}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\
\hline
\text{CS} & 1 & 1 & 1 & 0 & 0 \\
 & 2 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 \\
\text{MD} & 0 & 0 & 0 & 2 & 2 \\
 & 0 & 0 & 0 & 3 & 3 \\
 & 0 & 0 & 0 & 1 & 1
\end{array}
=
\begin{bmatrix}
0.18 & 0 \\
0.36 & 0 \\
0.18 & 0 \\
0.90 & 0 \\
0 & 0.53 \\
0 & 0.80 \\
0 & 0.27
\end{bmatrix}
\text{x}
\begin{bmatrix}
9.64 & 0 \\
0 & 5.29
\end{bmatrix}
\text{x}
\begin{bmatrix}
0.58 & 0.58 & 0.58 & 0 & 0 \\
0 & 0 & 0 & 0.71 & 0.71
\end{bmatrix}
$$

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

$$q= \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ 1 & 0 & 0 & 0 & 0 \end{array}$$

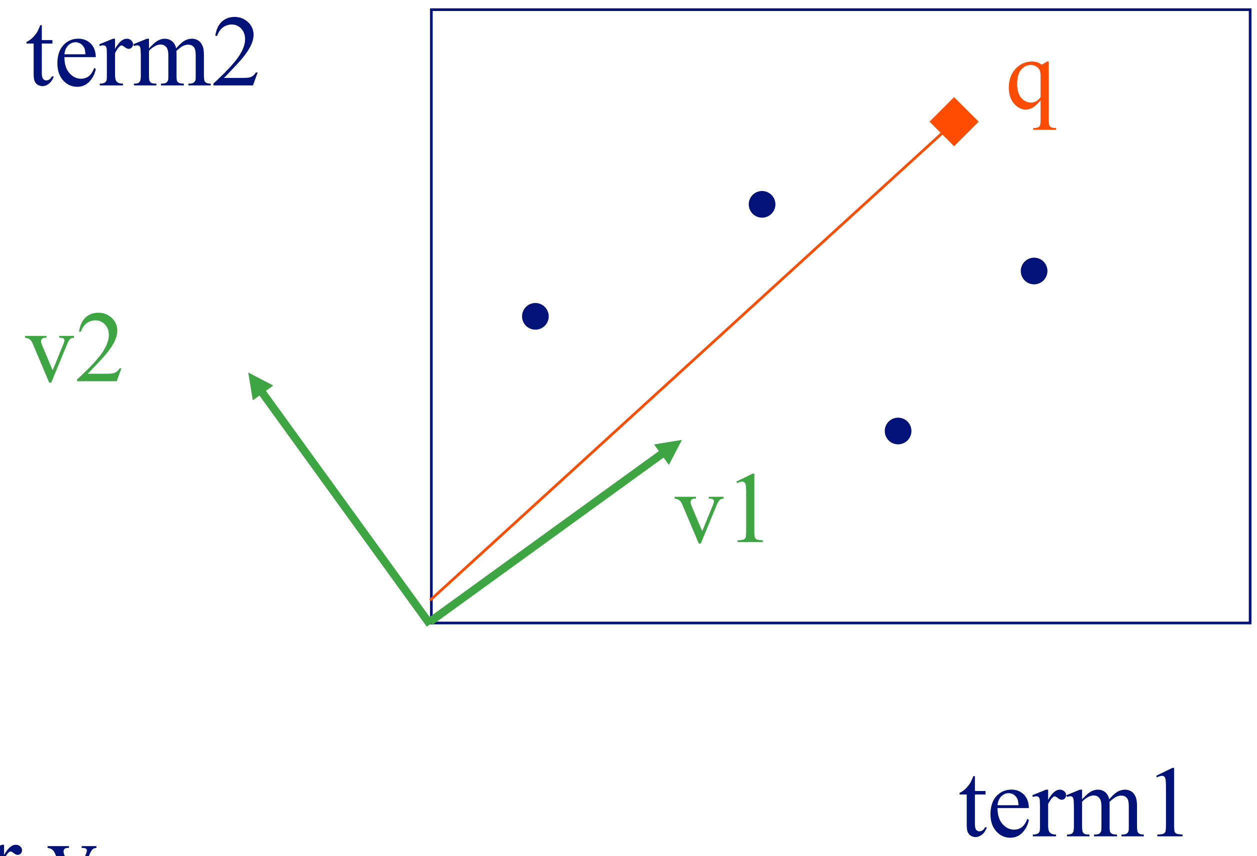


Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

$$q = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}$$



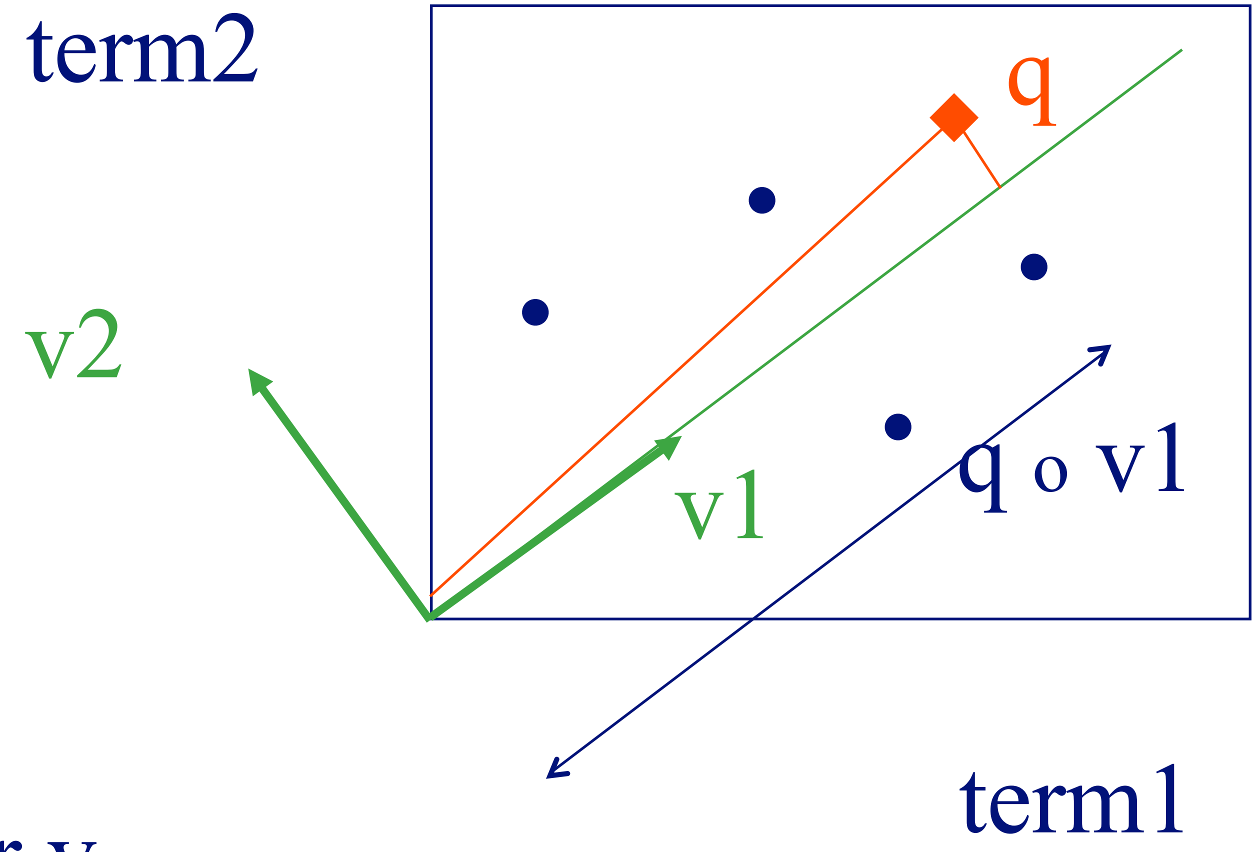
A: inner product (cosine similarity) with each ‘concept’ vector v_i

Case study - LSI

Q1: How to do queries with LSI?

A: map query vectors into ‘concept space’ – how?

$$q = \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}$$



A: inner product
(cosine similarity)
with each ‘concept’ vector v_i

Case study - LSI

compactly, we have:

$$q\ \mathbf{V} = q_{concept}$$

Eg:

CS-concept

$$q = \begin{array}{c} \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.71 \\ 0 & 0.71 \end{bmatrix} = \begin{bmatrix} 0.58 & 0 \end{bmatrix}$$

term-to-concept
similarities

Case study - LSI

Drill: how would the document (‘information’, ‘retrieval’) be handled by LSI?

Case study - LSI

Drill: how would the document ('information', 'retrieval') be handled by LSI? A: SAME:

$d_{concept}$ = d $\mathbf{V}$

Eg:

$$d= \begin{array}{ccccc} \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} \\ \hline 0 & 1 & 1 & 0 & 0 \end{array} \underbrace{\begin{bmatrix} 0.58 & 0 \\ 0.58 & 0 \\ 0.58 & 0 \\ 0 & 0.71 \\ 0 & 0.71 \end{bmatrix}}_{\text{term-to-concept similarities}} = \overset{\text{CS-concept}}{\begin{bmatrix} 1.16 & 0 \end{bmatrix}}$$

Case study - LSI

Observation: document ('information', 'retrieval') will be retrieved by query ('data'), although it does not contain 'data'!!

$$\begin{array}{ccccccccc} & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} & & \text{CS-concept} & \\ d= & 0 & 1 & 1 & 0 & 0 & \dashrightarrow & 1.16 & 0 \\ q= & 1 & 0 & 0 & 0 & 0 & \dashrightarrow & 0.58 & 0 \end{array}$$

Case study - LSI

Q1: How to do queries with LSI?

➡ Q2: multi-lingual IR (english query, on spanish text?)

Case study - LSI

- Problem:
  - given many documents, translated to both languages (eg., English and Spanish)
  - answer queries across languages

Case study - LSI

- Solution: ~ LSI

$$\left[\begin{array}{c|ccccc:ccccc}
 & \text{data} & \text{inf.} & \text{retrieval} & \text{brain} & \text{lung} & \text{datos} & \text{informacion} & & & \\
\text{CS} & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
 & 2 & 2 & 2 & 0 & 0 & 1 & 2 & 2 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
 & 5 & 5 & 5 & 0 & 0 & 5 & 5 & 4 & 0 & 0 \\
\text{MD} & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 2 & 2 \\
 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 2 & 3 \\
 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1
\end{array}\right]$$

Switch Gear to **Text Visualization**

What comes up to your mind?

What visualization have you seen before?

Word/Tag Cloud (still popular?)

http://www.wordle.net

Word Counts (words as bubbles)

http://www.infocaptor.com/bubble-my-page

Word Tree

word tree

We

☐ reverse tree ☐ one phrase per line

Shift-click to make that word the root.

we

must

act

,

we must act knowing that our work will be imperfect. We must act, knowing that today's victories will be only partial, and th

knowing that today's victories will be only partial, and that it will be up to those who stand here in four years, and forty years

knowing that our work will be imperfect. We must act, knowing that today's victories will be only partial, and that it will be up to those wh

do these things together, as one nation, and one people. This generation of Americans has been tested by crises that steeled our resolve and proved our resilience.

harness new ideas and technology to remake our government, revamp our tax code, reform our schools, and empower our citizens with the skills they need to wor

make the hard choices to reduce the cost of health care and the size of our deficit. But we reject the belief that America must choose between caring for the generat

lead it. We cannot cede to other nations the technology that will power new jobs and new industries – we must claim its promise.

claim its promise. That's how we will maintain our economic vitality and our national treasure – our forests and waterways; our croplands and snowcapped pea

carry those lessons into this time as well. We will defend our people and uphold our values through strength of arms and rule of law.

be a source of hope to the poor, the sick, the marginalized, the victims of prejudice – not out of mere charity, but because peace in our time requires the constant ac

faithfully execute that pledge during the duration of our service. But the words I spoke today are not so different from the oath that is taken each time a soldier sig

will

seize it – so long as we seize it together. For we, the people, understand that our country cannot succeed when a shrinking few do very well and a growing many barely m

respond to the threat of climate change, knowing that the failure to do so would betray our children and future generations.

maintain our economic vitality and our national treasure – our forests and waterways; our croplands and snowcapped peaks. That is how we will preserve our planet, c

preserve our planet, commanded to our care by God. That's what will lend meaning to the creed our fathers once declared.

defend our people and uphold our values through strength of arms and rule of law. We will show the courage to try and resolve our differences with other nations peacef

show the courage to try and resolve our differences with other nations peacefully – not because we are naïve about the dangers we face, but because engagement can mor

renew those institutions that extend our capacity to manage crisis abroad, for no one has a greater stake in a peaceful world than its most powerful nation.

support democracy from Asia to Africa; from the Americas to the Middle East, because our interests and our conscience compel us to act on behalf of those who long for

are

made for this moment, and we will seize it – so long as we seize it together. For we, the people, understand that our country cannot succeed when a shrinking few do very we

true to our creed when a little girl born into the bleakest poverty knows that she has the same chance to succeed as anybody else, because she is an American, she is free, and

also heirs to those who won the peace and not just the war, who turned sworn enemies into the surest of friends, and we must carry those lessons into this time as well.

naïve about the dangers we face, but because engagement can more durably lift suspicion and fear. America will remain the anchor of strong alliances in every corner of th

truly created equal, then surely the love we commit to one another must be equal as well. Our journey is not complete until no citizen is forced to wait for hours to exercise t

, the people,

still believe that

every citizen deserves a basic measure of security and dignity. We must

our obligations as Americans are not just to ourselves, but to all poster

enduring security and lasting peace do not require perpetual war. Our

understand that our country cannot succeed when a shrinking few do very well and a growing many barely make it.

declare today that the most evident of truths – that all of us are created equal – is the star that guides us still; just as it guided our forebea

have

. Through blood drawn by lash and blood drawn by sword, we learned that no union founded on the principles of liberty and equality could survive half-slave and half

never relinquished our skepticism of central authority, nor have we succumbed to the fiction that all society's ills can be cured through government alone.

always understood that when times change, so must we; that fidelity to our founding principles requires new responses to new challenges; that preserving our individ

lost, know too well the price that is paid for liberty. The knowledge of their sacrifice will keep us forever vigilant against those who would do us harm.

cannot

cede to other nations the technology that will power new jobs and new industries – we must claim its promise. That's how we will maintain our economic vit

walk alone; to hear a King proclaim that our individual freedom is inextricably bound to the freedom of every soul on Earth.

afford delay. We cannot mistake absolutism for principle, or substitute spectacle for politics, or treat name-calling as reasoned debate.

mistake absolutism for principle, or substitute spectacle for politics, or treat name-calling as reasoned debate. We must act, we must act knowing that our wo

all

define liberty in exactly the same way, or follow the same precise path to happiness. Progress does not compel us to settle centuries-long debates about the role of government for

make to the flag that waves above and that fills our hearts with pride. They are the words of citizens, and they represent our greatest hope.

gather to inaugurate a president, we bear witness to the enduring strength of our Constitution. We affirm the promise of our democracy.

bear witness to the enduring strength of our Constitution. We affirm the promise of our democracy. We recall that what binds this nation together is not the colors of our skin or the tenets of our fa

affirm the promise of our democracy. We recall that what binds this nation together is not the colors of our skin or the tenets of our faith or the origins of our names.

recall that what binds this nation together is not the colors of our skin or the tenets of our faith or the origins of our names.

hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable rights, that among these are Life, Liberty, and the pursuit of Happ

continue a never-ending journey, to bridge the meaning of those words with the realities of our time. For history tells us that while these truths may be self-evident, they have never been self-executi

learned that no union founded on the principles of liberty and equality could survive half-slave and half-free. We made ourselves anew, and vowed to move forward together.

made ourselves anew, and vowed to move forward together. Together, we determined that a modern economy requires railroads and highways to speed travel and commerce; schools and colleges

determined that a modern economy requires railroads and highways to speed travel and commerce; schools and colleges to train our workers.

discovered that a free market only thrives when there are rules to ensure competition and fair play. Together, we resolved that a great nation must care for the vulnerable, and protect its people fro

resolved that a great nation must care for the vulnerable, and protect its people from life's worst hazards and misfortune.

succumbed to the fiction that all society's ills can be cured through government alone. Our celebration of initiative and enterprise; our insistence on hard work and personal responsibility, these ar

; that fidelity to our founding principles requires new responses to new challenges; that preserving our individual freedoms ultimately requires collective action.

'll need to equip our children for the future, or build the roads and networks and research labs that will bring new jobs and businesses to our shores.

possess all the qualities that this world without boundaries demands: youth and drive; diversity and openness; an endless capacity for risk and a gift for reinvention.

seize it together. For we, the people, understand that our country cannot succeed when a shrinking few do very well and a growing many barely make it.

believe that America's prosperity must rest upon the broad shoulders of a rising middle class. We know that America thrives when every person can find independence and pride in their work; whe

know that America thrives when every person can find independence and pride in their work; when the wages of honest labor liberate families from the brink of hardship.

understand that outworn programs are inadequate to the needs of our time. We must harness new ideas and technology to remake our government, revamp our tax code, reform our schools, and e

reject the belief that America must choose between caring for the generation that built this country and investing in the generation that will build its future.

remember the lessons of our past, when twilight years were spent in poverty, and parents of a child with a disability had nowhere to turn.

do not believe that in this country, freedom is reserved for the lucky, or happiness for the few. We recognize that no matter how responsibly we live our lives, any one of us, at any time, may face a jo

recognize that no matter how responsibly we live our lives, any one of us, at any time, may face a job loss, or a sudden illness, or a home swept away in a terrible storm.

live our lives, any one of us, at any time, may face a job loss, or a sudden illness, or a home swept away in a terrible storm.

make to each other – through Medicare, and Medicaid, and Social Security – these things do not sap our initiative; they strengthen us.

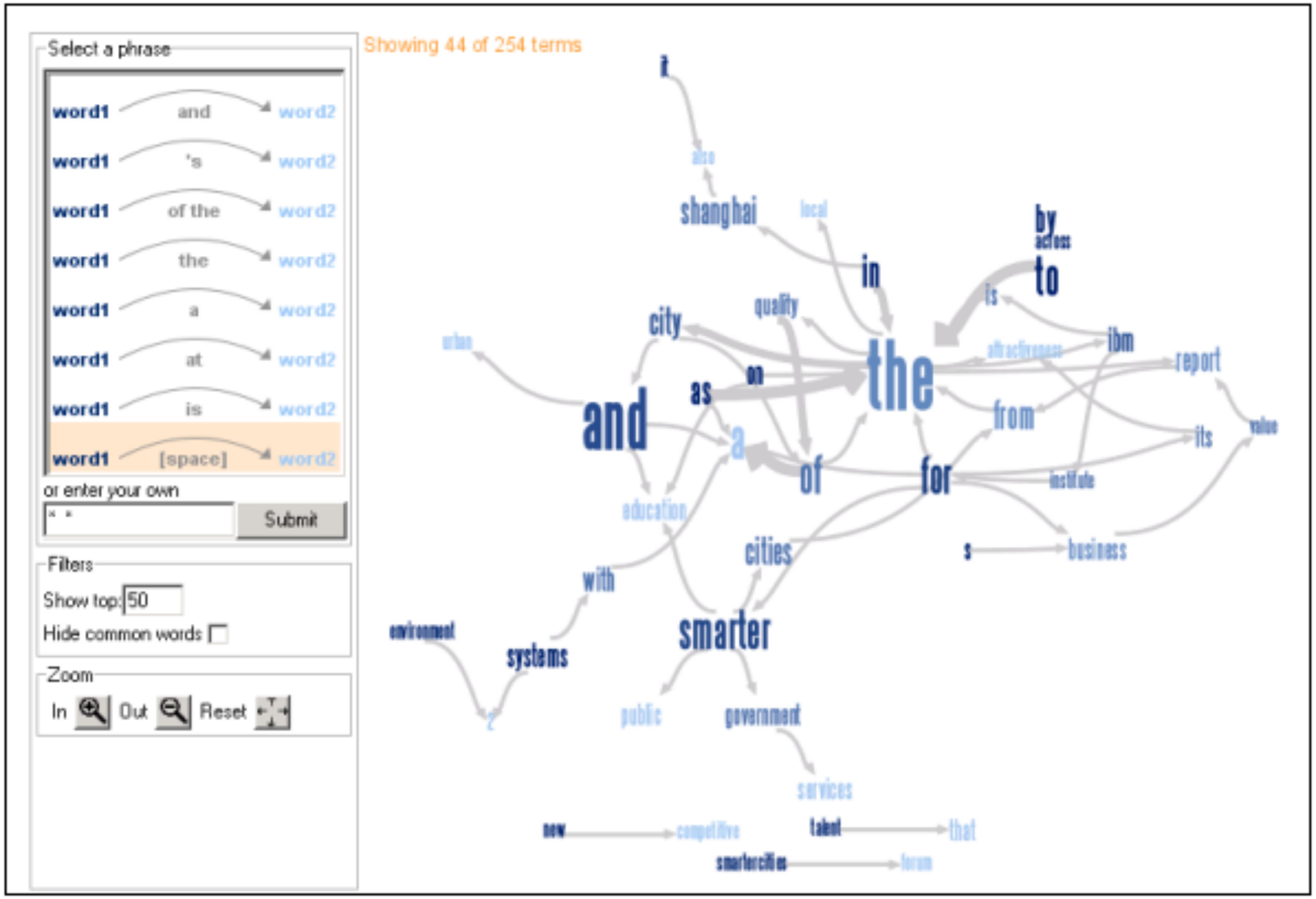
substitute spectacle for politics, or treat name-calling as reasoned debate. We must act, we must act knowing that our work will be imperfect. We must act, knowing that today's victories will be only partial, and that it will be up to those who stand here in four years, and forty years, and four hundred years hence to advance the timeless spirit once conferred to us in a spare Philadelphia hall.

My fellow Americans, the oath I have sworn before you today, like the one recited by others who serve in this Capitol, was an oath to God and country, not party or faction – and we must faithfully execute that pledge during the duration of our service. But the words I spoke today are not so different from the oath that is taken each time a soldier signs up for duty, or an immigrant realizes her dream. My oath is not so

http://www.jasondavies.com/wordtree/

Phrase Net

Visualize pairs of words that satisfy a particular pattern, e.g., X and Y



Show top: 50

Hide common words ☐

http://www-958.ibm.com/software/data/cognos/manyeyes/page/Phrase_Net.html