Graphs / Networks
Basics, how to build & store graphs, laws, etc.
Centrality, and algorithms you should know

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Georgia Tech

Partly based on materials by Professors Guy Lebanon, Jeffrey Heer, John Stasko, Christos Faloutsos, Le Song
Announcement: HW 1 & 2

Grades and feedback posted on T-Square

- Average score: 82-84 out of 90

Solution to be posted on course website

We aim to release HW 2 tomorrow (mostly on D3)
CSE Seminar (Re)Announcement
Friday (2/7), 11am-12pm, Klaus 1116 West

The Aha! Moment: From Data to Insight

Dafna Shahaf
Stanford University
(Graduated from CMU)

Very relevant to this class!
May give you project ideas.
Dafna is a faculty candidate; she’ll talk about some of her best work
Plus, 0.5% bonus point for attending.
Graphs (aka Networks)

- Basics, how to build graph, store graph, laws, etc.
- Centrality, scalable algorithms you need to know, how to visualize “large” graphs, challenges (research problems)
- Interactive tools to make sense of large graphs, applications, etc.
Internet
50 Billion Web Pages
Facebook
1.2 Billion Users

Modified from Marc_Smith, flickr
Citation Network
250 Million Articles

Modified from well-formed.eigenfactor.org
Many More

**Twitter**
Who-follows-whom (500 million users)

**Amazon**
Who-buys-what (120 million users)

**AT&T Cellphone Network**
Who-calls-whom (100 million users)

**Protein-protein interactions**
200 million possible interactions in human genome

# Large Graphs I Analyzed

<table>
<thead>
<tr>
<th>Graph</th>
<th>Nodes</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>YahooWeb</td>
<td>1.4 Billion</td>
<td>6 Billion</td>
</tr>
<tr>
<td>Symantec Machine-File Graph</td>
<td>1 Billion</td>
<td>37 Billion</td>
</tr>
<tr>
<td>Twitter</td>
<td>104 Million</td>
<td>3.7 Billion</td>
</tr>
<tr>
<td>Phone call network</td>
<td>30 Million</td>
<td>260 Million</td>
</tr>
</tbody>
</table>
How to **represent** a graph?

Conceptually, visually, programmatically, etc.?
How to represent a graph?

**Visually**

1 → 2 → 3 → 4

**Adjacency matrix**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Adjacency list**

1: 2, 3
2: 4
3: 2

**Edge list**

1, 2, 1 (most common distribution format)
1, 3, 3
2, 4, 2 (sometimes painful to parse when edges/nodes have many columns (some are text with double/single quotes, some are integers, some decimals, ...))
How to represent a graph?

Visually

<table>
<thead>
<tr>
<th>Source node</th>
<th>Target node</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2, 3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td></td>
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</tbody>
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Adjacency matrix

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<td>0</td>
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</tbody>
</table>

Adjacency list

1: 2, 3
2: 4
3: 2

Edge list

1, 2, 1
1, 3, 3
2, 4, 2
3, 2, 1

What do all these representations have in common?
How to represent a graph?

Visually

Adjacency matrix

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Adjacency list

1: 2, 3
2: 4
3: 2

Source node

1, 2, 1
1, 3, 3
2, 4, 2
3, 2, 1

Edge list

Each node is uniquely identified by a numeric ID.

Why?

How to assign an ID to a node?
Assigning an ID to a node

• Use a “map” (Java) / “dictionary” (Python) / SQLite

• Same concept: given an entity/node (e.g., “Tom”) not seen before, assign a number to it

• Here’s an example using SQLite

Hidden column; SQLite automatically created for you

<table>
<thead>
<tr>
<th>rowid</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tom</td>
</tr>
<tr>
<td>2</td>
<td>Sandy</td>
</tr>
<tr>
<td>3</td>
<td>Richard</td>
</tr>
<tr>
<td>4</td>
<td>Polo</td>
</tr>
</tbody>
</table>
How to use the node IDs?

You want to create an “index” for this column; then use “join”

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How to build graph edges?
Manually: Use SQL

You already did this in HW1

- e.g., find pairs of actors/actresses who have starred in the same movie
Use interactive tools

http://nodexl.codeplex.com

NodeXL

• Excel plugin
• Windows-only
Use interactive tools

http://www.cc.gatech.edu/gvu/ii/ploceus/

Ploceus: Network-based Visual Analysis of Tabular Data

- Zhicheng Liu, Sham Navathe, John Stasko. VAST 2011 (“Made in Georgia Tech”)
How to store "large" graphs?
How large is “large”? 

What do you think?

• In what units? Thousands? Millions?

How do you measure a graph’s size?

• Such as...

Highly subjective. And domain specific.
Storing large graphs...

On your laptop computer

• SQLite
• Neo4j (GPL license)

On a server

• MySQL, PostgreSQL, etc.
• Neo4j(?)

With a cluster (more details a few lectures down)

• Hadoop (generic framework)
• HBase(?) , inspired by Google’s BigTable
• Hama, inspired by Google’s Pregel
• FlockDB, by Twitter

• Comparison of “graph databases”
  http://nosql.mypopescu.com/post/40759505554/a-comparison-of-7-graph-databases
Storing large graphs on your computer

I like to use SQLite. Why?

• Easily handle up to gigabytes
  • Roughly tens of millions of nodes/edges (perhaps up to billions?). Very good! For today’s standard.

• Very easy to maintain: one cross-platform file

• Has programming wrappers in numerous languages
  • C++, Java (Andriod), Python, Objective C (iOS),...

• Queries are so easy!
  e.g., find all nodes’ degrees = 1 SQL statement

• Bonus: SQLite even supports full-text search
SQLite graph database schema

Simplest schema:

edges(source_id, target_id)

More sophisticated (flexible; lets you store more things):

```
CREATE TABLE nodes (
    id INTEGER PRIMARY KEY,
    type INTEGER DEFAULT 0,
    name VARCHAR DEFAULT '"
);
```

```
CREATE TABLE edges (
    source_id INTEGER,
    target_id INTEGER,
    type INTEGER DEFAULT 0,
    weight FLOAT DEFAULT 1,
    timestamp INTEGER DEFAULT 0,
    PRIMARY KEY(source_id, target_id, timestamp)));
```
Side note:

Full-Text Search (FTS) on SQLite

http://www.sqlite.org/fts3.html

Very simple. Built-in. Only needs 3 lines of commands.

- **Create** FTS table (index)
  
  ```
  CREATE VIRTUAL TABLE critics_consensus USING fts4 (consensus);
  ```

- **Insert** text into FTS table
  
  ```
  INSERT INTO critics_consensus SELECT critics_consensus FROM movies;
  ```

- **Query** using the “match” keyword
  
  ```
  SELECT * FROM critics_consensus WHERE consensus MATCH 'funny OR horror';
  ```

Originally developed by Google engineers
Project idea

• Compare scalability between SQLite, Neo4j, HBase, etc.

• Which uses more space? What’s the maximum graph size?

• Which answers queries the fastest? For what queries? How does that change with the graph size?
I have a graph dataset. Now what?

Analyze it! Do “data mining” or “graph mining”.

How does it “look like”? Visualize it if it’s small.

Does it follow any expected patterns? Or does it *not* follow some patterns (outliers)?

• Why does this matter?

• If we know the patterns (models), we can do prediction, recommendation, etc.
  e.g., is Alice going to “friend” Bob on Facebook? People often buy beer and diapers together.

• Outliers often give us new insights
  e.g., telemarketer’s friends don’t know each other
Finding patterns & outliers in graphs

Outlier/Anomaly detection (will be covered later)

• To spot them, we need to patterns first
• Anomalies = things that do not fit the patterns

To effectively do this, we need large datasets

• patterns and anomalies don’t show up well in small datasets
Are real graphs random?

Random graph (Erdos-Renyi)
100 nodes, avg degree = 2

No obvious patterns

Generated with pajek
http://vlado.fmf.uni-lj.si/pub/networks/pajek/
Graph mining

• Are real graphs random?
Laws and patterns

• Are real graphs random?
• A: NO!!
  – Diameter (longest shortest path)
  – in- and out- degree distributions
  – other (surprising) patterns

• So, let’s look at the data
Power Law in Degree Distribution

- Faloutsos, Faloutsos, Faloutsos [SIGCOMM99]
  Seminal paper. Must read!

\[ \log(\text{rank}) \]

\[ \log(\text{degree}) \]

internet domains

att.com

ibm.com

log(degree)

log(rank)
Power Law in Degree Distribution

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\[ \log(\text{rank}) \]
\[ \log(\text{degree}) \]

internet domains

\[ \text{att.com} \]
\[ \text{ibm.com} \]

\[ \log(\text{degree}) \]

-0.82

\[ \log(\text{rank}) \]
Power Law in Eigenvalues of Adjacency Matrix

Eigen exponent = slope = -0.48

Eigenvalue vs Rank of decreasing eigenvalue
How about graphs from other domains?
More Power Laws

- Web hit counts

[Alan L. Montgomery and Christos Faloutsos]

Web Site Traffic

log(#website)

log(#website visit)

log(#website visit)

users

sites

ebay
epinions.com

- who-trusts-whom [Richardson + Domingos, KDD 2001]

(out) degree

trusts-2000-people user
And numerous more

• # of sexual contacts
• Income [Pareto] – 80-20 distribution
• Duration of downloads [Bestavros+]
• Duration of UNIX jobs
• File sizes
• …
Any other ‘laws’?

• Yes!

• Small diameter (~ constant!) –
  • six degrees of separation / ‘Kevin Bacon’
  • small worlds [Watts and Strogatz]
Problem: Time evolution

• Jure Leskovec (CMU -> Stanford)
• Jon Kleinberg (Cornell)
• Christos Faloutsos (CMU)
Evolution of the Diameter

• Prior work on Power Law graphs hints at slowly growing diameter:
  • diameter $\sim O(\log N)$
  • diameter $\sim O(\log \log N)$

• What is happening in real data?
Evolution of the Diameter

• Prior work on Power Law graphs hints at slowly growing diameter:
  • diameter $\sim O(\log N)$
  • diameter $\sim O(\log \log N)$
• What is happening in real data?
• Diameter shrinks over time
Diameter – “Patents”

- Patent citation network
- 25 years of data
- @1999
  - 2.9 M nodes
  - 16.5 M edges
Temporal Evolution of the Graphs

- $N(t)$ ... nodes at time $t$
- $E(t)$ ... edges at time $t$
- Suppose that
  - $N(t+1) = 2 \times N(t)$
- Q: what is your guess for
  - $E(t+1) =? 2 \times E(t)$
Temporal Evolution of the Graphs

- $N(t)$ … nodes at time $t$
- $E(t)$ … edges at time $t$
- Suppose that
  \[ N(t+1) = 2 \times N(t) \]
- Q: what is your guess for
  \[ E(t+1) =? 2 \times E(t) \]
- A: over-doubled!

But obeying the "Densification Power Law"
Densification – Patent Citations

- Citations among patents granted
- @1999
  - 2.9 M nodes
  - 16.5 M edges
- Each year is a datapoint

\[
\begin{align*}
N(t) & \propto t^{1.66} \\
E(t) & = 0.0002 \times t^{1.66} \quad (R^2=0.99)
\end{align*}
\]
So many laws!

There will be more to come...

To date, there are 11 (or more) laws

- RTG: A Recursive Realistic Graph Generator using Random Typing [Akoglu, Faloutsos]

What should you do?

- **Try as many distributions as possible** and see if your graph fits them.
- **If it doesn’t, find out the reasons**. Sometimes it’s due to errors/problems in the data; sometimes, it signifies some new patterns!
Polonium: Tera-Scale Graph Mining and Inference for Malware Detection [Chau, et al]